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Numbers in Elementary Propositions: Some remarks on writings before and after *Some Remarks on Logical Form*

Abstract

It is often held that Wittgenstein had to introduce numbers in elementary propositions due to problems related to the so-called colour-exclusion problem. I argue in this paper that he had other reasons for introducing them, reasons that arise from an investigation of the continuity of visual space and what Wittgenstein refers to as 'intensional infinity'. In addition, I argue that the introduction of numbers by this route was prior to introducing them *via* the colour-exclusion problem. To conclude, I discuss two problems that Wittgenstein faced in the writings before *Some Remarks on Logical Form* (1929), problems that are independent of the colour-exclusion problem but dependent on the introduction of numbers in elementary propositions.

1. Numbers in Some Remarks on Logical Form

In *Some Remarks on Logical Form* (RLF) Wittgenstein wrote:

> I wish to make my first definite remark on the logical analysis of actual phenomena: it is this, that for their representation numbers (rational and irrational) must enter into the structure of the atomic propositions themselves. (RLF: 165)

In the same article, Wittgenstein justifies the need for introducing numbers in propositions consisting of an assignment of a degree of a property that admits gradation (e.g. colour) to a certain object by showing (or rather arguing very briefly) the impossibility of an analysis in terms of a logical product and a "completing supplementary statement". Given the fact that each degree of a quality excludes every other, Wittgenstein was led to abandon one of the cornerstones of the *Tractatus*, the thesis of the logical independence of elementary propositions.

While some commentators have found Wittgenstein's arguments for the unanalysability of statements of degree cogent and even obvious,[1] others have raised doubts concerning the force of the argument vis-à-vis the Tractarian background.[2] I shall not revisit this issue here but would point out that the introduction of numbers in elementary propositions does not occur only within the context of ascriptions of properties that admit gradation. Numbers were already used by Wittgenstein in passages pre-RLF and even in RLF itself to demarcate a place in a space (e.g. the visual space). In this connection, I shall argue that there is strong evidence in pre- (and post-) RLF passages (from MS 105-106 as presented in the *Wiener Ausgabe*)[3] for taking Wittgenstein to have had other reasons for introducing numbers in elementary propositions, reasons independent of the colour-exclusion problem.[4]

I begin by taking a closer look at the role played by numbers in RLF. After stating that numbers must enter into the structure of atomic[5] propositions, Wittgenstein asks the reader to imagine a

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[1] See Hacker 1986: 108-9; see also Marion 1998: 123.

[2] See Ricketts 2014; see also Lugg 2015.

[3] Wittgenstein's manuscripts were transcribed and put into chronological order in the *Wiener Ausgabe* edition, from which I shall quote the relevant passages. In the manuscripts, Wittgenstein used first the recto and then the verso pages and for this reason the order of the pages is not the order in which they were written. The first volume of the *Wiener Ausgabe* edition (hereafter Wi1) contains the remarks made in MS 105 and 106 (for the most part undated). The material from the manuscripts that covers the text of RLF (1929) is found at MS 106 pp. 71–111 (Wi1, pp. 55–63), cf. *Wittgenstein Source* <*http://www.wittgensteinsource.org/BFE/Ms-106_f*>. By pre- and post-RLF passages I mean the passages written before and after these pages. For information about the chronological order of MS 105-6, see Engelmann 2013.

[4] I am using the expression "colour-exclusion problem" to refer to the general issue (not limited to colours) of ascriptions of properties that admit gradation.

[5] The term "atomic proposition" is used by Russell and is equivalent to the tractarian "elementary proposition".

system of rectangular axes drawn in the visual field together with an arbitrarily fixed scale (in short, a coordinate system). He continues:

> It is clear that we then can describe the shape and position of every patch of colour in our visual field by means of statements of numbers which have their significance relative to the system of co-ordinates and the unit chosen. Again, it is clear that this description will have the right logical multiplicity, and that a description which has a smaller multiplicity will not do. (RLF: 165)

He then gives the example of the use of such a coordinate system to describe a patch and attribute to it the colour red. He takes the proposition to be symbolized as "[6–9, 3–8] R" and argues that the unanalysed term "R" must contain numbers when properly analyzed inasmuch as a specific degree of red is being assigned to the patch. It is tempting to think that an examination of this proposition suffices to show that numbers are already present in elementary propositions. After all, "[6–9, 3–8]" includes numbers, these numbers being used to designate the "object" of which red is predicated. Since "R" is the only "unanalyzed term", an analysis of this proposition, whatever it may be, will have to include numbers in elementary propositions. It is hard to see, then, why Wittgenstein needs to show that "R" too includes numbers. This temptation should be resisted, however. For it may be the case that the representation of a place in visual space by means of numbers is a merely feature of a particular symbolism. That it is not, i.e. that it is, as Wittgenstein puts it, "an essential and, consequently, unavoidable feature of the representation" (RLF: 166) has to be justified.

I take the argument that Wittgenstein presents in the fourth paragraph of RLF as a justification to introduce numbers in statements expressing the degree of a quality. But it is important to note that, although the quality that is assigned to the patch "[6–9, 3–8]" admits gradation, the patch itself is not the degree of any quality. The reason for this is that the description of the spatial characteristics of a patch in visual space (its size and position) is not a (true/false) proposition at all, the patch being identifiable only by its size and position. That is, the size and position of a patch in visual space are not (external) *properties* of a thing that can be independently identified inasmuch as the size and position of a patch *constitute* the patch. The ascription of a colour to a patch in visual space, by contrast, is a proposition, because the object to which the colour is predicated is identified independently of its colour, namely, by its size and position. Therefore, the numbers appearing in the symbol "[6–9, 3–8]" do not refer to the degree of any quality that would be ascribed to a thing. In this case, the problem of how to analyze *propositions expressing the degree of a quality* simply does not exist.[6]

Because of this asymmetry, I do not take the argument that Wittgenstein presents in the fourth paragraph of RLF as providing

[6] There is a passage in WWK: 75 which seems to contradict what I am saying. In this passage Wittgenstein apparently treats the description of a rectangle by giving its coordinates and the description of its colour as having the same status. The most problematic sentence from this passage is the following: "Jedes Rechteck kann ich beschreiben durch vier Zahlenangaben, nämlich durch die Koordinaten des linken oberen Eckpunktes, durch seine Länge und durch seine Breite, also durch (x, y; u, v). Die Angabe dieser vier Koordinaten ist mit jeder anderen Angabe unverträglich. Ebenso ann ich die Farbe des Rechtecks beschreiben, indem ich gleichsam die Farbenskala anlege". The idea would be that two spatial specifications (x_1, y_1; u_1, v_1) and (x_2, y_2; u_2, v_2) are *incompatible* because they are specifications of the *same* rectangle (they predicate incompatible properties of the same substrate). I cannot see, however, how the identity of a rectangle, in the context of a complete description of visual space, could be given independently of its size and position. For the size and position of a thing can only be *predicated* of it if they are not criteria for identifying it, if they are *external* properties of it (i.e., if it is thinkable that the thing does not possess this property). In cases like "red is a colour", where it seems that an *internal* property is being predicated of a thing, what we actually have is not a *proposition*, but the specification of a variable's value (see PB: I-3b). So it seems that in the above passage Wittgenstein is simplifying the matter to make a point (in the broader context of the conversation) that is independent of the distinction I am drawing. This distinction, however, is present in some passages where Wittgenstein considers the matter more thoroughly. For instance, in PB: IX-96b, he says that "red" and "circle" are not "properties" that are on the same level, for "it is easy to imagine *what* is red but difficult to imagine what is circular". He goes on to say that "the position is part of the form". In other words, position is a formal (internal) property, not a material (external) one, and this is the point I am stressing. It is worth also taking a look at WWK: 54, where Wittgenstein says that "von diesen zwei bestimmten Strecken ist es freilich nicht denkbar, daß die eine länger oder kürzer ist als die andere". The idea here is that since the length of a line segment is an internal property of it, it does not makes sense to *say* it is longer or shorter than another given line segment. This would show itself in the symbolism for representing these line segments. I would like to thank an anonymous referee for drawing my attention to the passage of WWK: 75.

a reason for thinking that numbers should occur in symbols for patches in visual field. Moreover, the introduction of numbers in the case of propositions about the size and position of patches in visual field would not have the same immediate consequence as in the case of ascriptions of degrees of a quality.[7] Whereas the introduction of numbers in statements expressing the degree of a quality immediately implies that elementary propositions exclude one another, it does not follows immediately that the introduction of numbers to specify patches in visual field implies that elementary propositions are not logically independent. Why should two propositions "[6–9, 3–8]R" and "[2–3, 1–2]R" exclude one another? There is no easy way here to argue for the existence of logical relations between elementary propositions.

So, in the end, the only reason we find in RLF for introducing numbers to describe patches in visual field is that "a description which has a smaller multiplicity will not do", and we are left with the task of explaining *what* is a description which has a smaller multiplicity and *why* this kind of description does not work. This double task, I argue, is necessary to understand the underlying reasons Wittgenstein had to introduce numbers to represent visual phenomena. In this paper I shall address these two questions by considering some remarks in pre- (and post-) RLF writings.

2. The introduction of the "expansible sign"

In the first few pages of MS 105 Wittgenstein investigates the possibility of a phenomenological description of visual space. By this I mean a description of the structural properties of visual space (these properties would then be shown in the symbolism for describing visual phenomena). The direction of his investigation does not follow a precise route, and he skips from topic to topic. Nevertheless it can be safely said that the analysis of visual space is at the centre of his attention. It is clear that he takes the description of a patch in visual space to involve the analysis of the visual space as a whole, the place occupied by the patch being a sub-region of visual space. Furthermore the specification of a patch does not

[7] A similar point is made in passing in Ferraz Neto 2003: 108.

require the actual existence of the patch (i.e., the fact that the region specified corresponds to the area of a monochromatic patch), but only its possibility. The analysis of visual space must furnish the domain of possibilities to which the specification of a patch belongs. This is done by constructing a symbolism with a particular mathematical multiplicity.

The first key passage I would like to consider in the 1929 manuscripts is on page 55 of MS 105. There Wittgenstein mentions the need for introducing something like an expansible (*dehnbar*) sign to represent space. He writes:

> In order to represent space we need – so it appears to me – something like an expansible sign.
>
> A sign that makes allowance for an interpolation, similar to the decimal system.
>
> The sign must have the multiplicity and properties of space. (MS 105: 55; Wi1: 15; PB: XVI-177c)

The allusion here is to the property of the numeral system of "decimal places" (used to represent rational numbers), according to which it is possible to build, from two given numerals, a completely new numeral, an "interpolation" (i.e. a numeral expressing a number that is between two given numbers). Thus if, in a given system of spatial coordinates, a patch is specified in some dimension by the interval "[0–1]", the very sign, together with the rules for handling the numeral system, shows the possibility of having, e.g., two smaller patches of the same size occupying the same place as the first patch: "[0–0.5]" and "[0.5–1]". Notice that, in virtue of the density of the rationals, this process could continue *ad infinitum*.[8]

These entries, which Wittgenstein will include in Chapter XVI of *Philosophische Bemerkungen* (PB), are followed by the following question: "Isn't the decimal system with its infinite possibility of interpolation precisely this sign?" The remarks about the subject of

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[8] The set of rational numbers with the standard ordering is a densely ordered set, in the sense that for every two rational numbers x and y such that $x < y$, there is a rational number z such that $x < z < y$.

infinity, on the other hand, were later incorporated to Chapter XII of PB. One of these is the remark that immediately follows the last question:

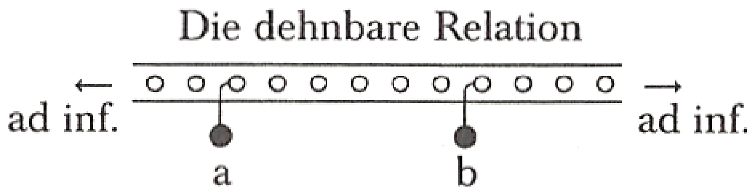
> The rules for a number system – say, the decimal system – contain everything that is infinite about the numbers. That, e.g., these rules set no limits on the left or right hand to the numerals; *this* is what contains the expression of infinity.
>
> Someone might perhaps say: True, but the numerals are still limited by their use and by writing materials and other factors. That is so, but that isn't expressed in the rules for their use, and it is only in these that their real essence is expressed. (MS 105: 55-7; Wi1: 16; PB: XII-141a)

The introduction of the expansible sign and "intensional infinity"[9] continue to be discussed in tandem throughout the manuscripts. Besides the representation of the size and position of patches, Wittgenstein thought that an expansible sign is necessary to represent distance relations between patches in visual field. In this connection, he remarks that the concept of distance in visual field is not to be confused with the concept of physical distance that arises by means of the stipulation of a rigid ruler, but instead must arise immediately from the structure of visual field:

> A spatial distance can be represented by a number. (This proposition is not concerned with rigid rulers). It must arise immediately from the structure of visual space.
>
> Instead of writing the spatial relation of two patches a and b as "aRb", I could then write aNb, where N is a number, therefore an *expansible* relation. (MS 105: 98-100; Wi1: 26)

The connection between the expansible sign and the subject of "infinite possibility" shows itself again when the "expansible relation" is pictured on page 51 of MS 106 (Wi1: 50) with the following image:

9 By "intensional infinity" I mean "infinity as a property of a rule" as opposed to "infinity as a property (cardinality) of an extension".



It is not altogether clear, however, what this picture means. The most reasonable interpretation is that the right boundary and the left boundary of the patches *a* and *b* are being marked in the left of the small circles representing ticks of a ruler. The image above would then represent the circumstance that the patch *a* is 6 units far from the patch *b*. The "expandability" of the relation seems to follow by the following reasoning: since it is always possible to consider a new unit with, e.g., the half of the size of the old unit, at some point the figure will eventually become unreadable (the ticks will overlap, the space between the ticks will vanish, etc.) and the symbol will have to be stretched to represent the units properly. On this interpretation, the arrows to the left and to the right indicate that there are no limits to this stretching. That is, it is always possible to consider an even smaller unit in our symbolism and we are not bounded by spatial restrictions any more than numerals are bound by digits to the left or to the right.

It is clear that rational numbers are expansible inasmuch as they constitute a densely ordered set. It is, however, not altogether clear that every expansible sign has to be a *number*. Numbers seem to introduce more than density, namely, a distance relation. But if the concept of distance applies to the structure of visual field (as Wittgenstein thought at that time), then it must be present in the symbolism for representing visual phenomena. In this case, then, the expansible signs needed to represent space have to be numbers.

In the next section, I hope to show how the connection between the introduction of the expansible sign and the subject of infinity suggests an answer to the two questions I raised in the Introduction, namely, what is a description (of visual space) with a smaller multiplicity (than the one that makes use of numerical coordinates) and why does this kind of description not work.

3. Infinity and the analysis of visual space

It is indisputable that the subject of infinity plays a major role in the writings of the so-called "middle period" of Wittgenstein's philosophical development. However, it is not entirely clear what precisely this role is, more specifically what is at stake and why he works on the issue. I shall argue that the discussion of intensional infinity plays a major role in writings pre-RLF because it led Wittgenstein to abandon the discreteness of simple objects (one of the main features of the Tractarian ontology). First of all, I would like to draw attention to a post-RLF passage in MS 106, in which Wittgenstein asks if it is really necessary to introduce the expansible sign to represent space. Although this passage is located after the preparatory writings for the composition of RLF, it is clear from it and from our previous discussion that Wittgenstein is referring retrospectively to issues that appeared already in MS 105. The passage reads as follows:

> But now one could ask: are those signs with infinite possibility really necessary; doesn't it work with the disjunction of the smallest visible parts? No. Because with the signs for the discrete parts the continuity could not be represented. – And what about the infinite possibility of the future? Why must it be expressed in propositions about temporal things? Because no matter how long I assume the future to be, an even longer future must be *able* to be assumed.
>
> The possibility of the finite is simply *without* end. (MS 106: 213; Wi1: 148)

This passage strongly suggests a relation between the problem of representing the continuity of space and the introduction of the expansible sign (the "sign with infinite possibility").[10] But, more than that, it appears to provide an answer to our questions. When Wittgenstein says in RLF that the description using numbers has the right logical multiplicity and a description which has a smaller multiplicity will not do, he is certainly thinking of the issue of the continuity of visual space. That is, a description which has a "smaller multiplicity" is a description of visual space built on discrete signs (signs for *minima visibilia*). Numbers (expansible signs)

[10] See also Wi1: 104: "The Continuum is quite inconceivable with discrete concepts".

are needed because the description by means of discrete signs does not have the "right multiplicity" for representing visual phenomena. Thus, a symbolism which has a smaller multiplicity will not do because such a symbolism it incapable of representing the continuity of visual space.

The issue of continuity of visual space reappears several times in the MS 105-108. One such time was immediately after having finished the composition of PB in 1930 and returned to Cambridge after a few days in Austria. The problem Wittgenstein was facing at that occasion is how to analyze the proposition: "the patch A is between the two limits B and C":

> If one says that the patch A is somewhere between the limits B and C, isn't it obviously possible, then, to describe or to depict a number of
>
> 
>
>
> positions of A between B and C, so that I see the succession of all these positions as a continuous transition? And the proposition that A is somewhere between B and C is not precisely the disjunction of all these N positions?
>
> But what about these N pictures? It is clear that two immediately following pictures cannot be visually distinguishable otherwise the transition would be visually discontinuous. (Ms 108: 134-5; Wi2: 242)

Here again there is a tension between the possibility of pointing out a set of discrete positions where a patch might be located and the continuity of visual phenomena. And at some point (see *infra*), Wittgenstein must have come to the conclusion that the continuity of visual space implies the impossibility of specifying a number of discrete positions or parts that would supposedly compose the visual space, these discrete parts being the smallest visible differences between two spatial intervals.

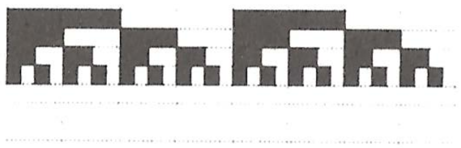
I take the specification of these discrete parts of space to be implied by the task of the application of logic that Wittgenstein attempts to settle in the *Tractatus*. That is, I take this task as a continuation of the Tractarian project of providing, by means of an analysis of ordinary language, which are the names that compose elementary propositions, i.e. the signs in the language that are not composite. These discrete parts (the "simple parts" of visual phenomena) would not be, of course, obtained by physicalist experiments (i.e., by independent criteria) but would be given by the structure of visual field:

> The smallest visible difference would be one that would carry *in itself* the criterion of being the smallest.
>
> For in the case of the patch A between B and C, we simply distinguish some positions and do not distinguish others. What we needed, however, was so to speak an infinitesimal difference, therefore a difference that would carry in itself that it is being the smallest. (MS 108: 135-6; Wi2: 243)

But now Wittgenstein recognizes this idea is absurd, and sticks with his conviction that space is not composed by discrete parts at all:

> Space is evidently *not* composed of determinate (discrete) parts.
>
> For otherwise it should be possible to say immediately which are these parts.
>
> Space, however, is evidently homogeneous. (MS 108: 136; Wi2: 243)

This conclusion about the non-discrete character of space was already in some pre-RLF passages from MS 105, as I shall now show. In this manuscript, problems regarding the smallest visible difference appear for the first time when Wittgenstein was dealing with the subject of colours, more specifically, with the possibility of a metric for colours. Some of these remarks were later added to paragraph 218 (Chapter XXI) of PB; in MS 105 they are followed by passages about infinity and continuity, passages in turn added to paragraphs 136-7 of PB (Chapter XII). I quote these passages in full:

> If I have a series of alternately black and white patches, as shown in
>
> 
>
> the diagram then by continual bisection, I will soon arrive at a limit where I'm no longer able to distinguish the black and white patches, that is, where I have the impression of a grey strip.
>
> But doesn't that imply that a strip in my visual field cannot be bisected indefinitely often? And yet I don't see a discontinuity and of course I

> wouldn't, since I could only see a discontinuity if I hadn't yet reached the limit of divisibility.
>
> This seems very paradoxical.
>
> But what about the continuity between the individual rows? Obviously we have a last but one row of distinguishable patches and then a last row of uniform grey; but could you tell from this last row that it was in fact obtained by bisecting the last but one? Obviously not. On the other hand, could you tell from the so-called last but one row that it can no longer be visibly bisected? It seems to me, just as little. In that case, there would be no last visibly bisected row!
>
> If I cannot visibly bisect the strip any further, I can't even try to, and so can't see the failure of such an attempt. (This is like the case of the limitlessness of visual space.)
>
> Obviously, the same would hold for distinctions between colours.
>
> Continuity in our visual field consists in our not seeing discontinuity. (MS 105: 92-4; Wi1: 25; PB: XII-137)

The reasoning here is that when a patch is repeatedly subdivided into sub-patches as in the Figure, we effectively reach a stage where we do not see a number of discrete and distinct parts, but only a single grey patch. The preceding stage, however, does not provide the discrete elements that would compose the visual space, since we only see the strip composed of discrete elements if the limit of divisibility has not yet been reached, i.e., if there is the possibility of a further division. What happens in this case is that the grey patch is not recognized as the result of the bisection (as Wittgenstein puts it: "could you tell from this last row that it was in fact obtained by bisecting the last but one? Obviously not."), and not that the patches that compose the preceding stage cannot be further divided. In this sense, it is futile to search for the discrete elements that would constitute space since no matter how further the space is subdivided, no subdivision appears as the last possible subdivision.

In MS 106 Wittgenstein returns to this example and draws essentially the same conclusion:

> Just think of the black and white striped field with the thinnest stripes we still can see. Are these for us the indivisible simple elements of the

> visual field? No. We recognize them as divisible, but not divided. (MS 106: 205; Wi1: 146)

He then continues with some remarks on the subjects of infinite, reality and possibility. Most of these remarks were incorporated in paragraph 139 of Chapter XII of PB. The upshot of the discussion is simply that the meaning of the infinite divisibility of space is actually that "space is not composed of singular things (parts)" (MS 106: 147; PB: XII-139d). The description of a monochrome patch, for instance, is not the conjunction of descriptions of tiny spatial parts that would compose the patch. Similarly, the statement that the patch A is somewhere between the limits B and C is not the disjunction of some discrete positions that the patch A could occupy between B and C. The length of an interval is not measured by the number of discrete parts contained in it and propositions describing patches (and relations between them) in visual space are not truth-functionally built from propositions describing discrete and ultimate elements of this space.

With the abandonment of spatial atomism, Wittgenstein considered it necessary to introduce expansible signs to represent space, signs which, together with the rules for their employment, could be used to represent every possible configuration of patches in visual space, including their position, size and relative distance. Since the concept of distance is intrinsic to visual space, these expansible signs are *numbers*, and the symbolism to represent space has the multiplicity of a *numeral system*.

4. Priority issues and conclusion

RLF was to be presented in a meeting of the Aristotelian Society on July 13, 1929. In the event, however, Wittgenstein chose to discuss topics related to infinity in mathematics, rather than presenting the written text. In the preceding sections, I have argued that issues about infinity were intimately related to the introduction of numbers in elementary propositions to specify the position of (and relations between) patches in visual space, and this result is presupposed (and not argued for) in RLF. What is argued for (again, very briefly) in RLF is that numbers have also to occur in propositions that ascribe the degree of a property that admits gradation to some object, and for this reason the thesis of the logical independence of elementary propositions is untenable. In this section, I give further evidence for the fact that the introduction of numbers via the discussion of infinity is prior to the same conclusion via the colour-exclusion problem.

As noted above (cf. fn. 3), the material from the manuscripts that covers the text of RLF is found at MS 106 pp. 71–111 (Wi1, pp. 55-63). Now I would add that Wittgenstein had come by page 71 to the conclusion that the description of the configurations of patches in visual space by means of numbers has the right multiplicity:

> The multiplicity of the spatial description is given by the fact that the description has the right multiplicity if it is capable of describing all thinkable configurations.
>
> Thus if one can describe the space in all its possibilities with sentences of the kind φ(m–n), then the description is in order and more is not needed. (MS 106: 69-71; Wi1: 55)

As in RLF, the symbol "m–n" denotes the continuous interval between the numbers m and n. This passage is followed in the manuscripts by the long paragraph that opens Chapter VIII of PB, the chapter in which the colour-exclusion problem is discussed at length. This paragraph ends with the following remarks:

> That makes it look as if a construction might be possible within the elementary proposition. That is to say, as if there were a construction in logic which didn't work by means of truth functions.
>
> But now it also seems, additionally, that these constructions have an effect on one proposition's following logically from another.
>
> For, if different degrees exclude one another it follows from the presence of one that the other is not present. In that case, two elementary propositions can contradict one another. (MS 106: 75; Wi1: 56; PB: VIII-76c)[11]

The last passage summarizes the colour-exclusion problem. In manuscript 106, there is a sentence between the first and the

[11] I modified the translation of the second paragraph. The original sentence is as follows: "Nun aber scheint es außerdem, daß diese Konstruktionen eine Wirkung auf das logische Folgen eines Satzes aus einem anderen haben".

second paragraph of this passage. It reads: "I have already wanted to say this with my relations that are expressed by numbers". Wittgenstein is referring to the distance relation between patches mentioned earlier (it is the only relation expressed by numbers to be found earlier in MS 105-6). It is also clear from the second paragraph that the logical exclusion between elementary propositions is an *additional effect* that occurs in this specific case of constructions "within the elementary proposition", and not an immediate consequence of every non-truth-functional construction. In other words, the abandonment of the idea of completeness of truth-functional constructions and the abandonment of the idea that elementary propositions are logically independent are not just two sides of the same coin. That is why, when numbers were introduced to represent patches in visual space, this unwelcome conclusion was not immediately drawn as a simple corollary. It is thus false to say that the conclusion that elementary propositions are not logically independent *allowed* the introduction of numbers at the elementary level (compare Engelmann 2013: 11). Wittgenstein does not introduce numbers in elementary propositions as a consequence of giving up the search for a Tractarian solution to the colour-exclusion problem.

It is true that Wittgenstein had already touched on the subject of colour-exclusion long before page 71 of MS 106: remarks on this subject started on p. 70 of MS 105 (Wi1, p. 22). This is, however, not a problem for my argument. To begin with, the idea of using numbers to represent space occurs even earlier in the manuscripts. Moreover, none of these earlier remarks on the colour-exclusion problem entail that Wittgenstein had already given up the search for a set of logically independent propositions. In fact, the first two main problems that Wittgenstein faces in the writings pre-RLF, namely, the status of arithmetic and the determinateness of sense, are independent of the colour-exclusion problem. They spring from the introduction of numbers to represent configurations of patches in visual space. To conclude I shall explore briefly these two problems.

The first problem has to do with a certain incompatibility between the Tractarian conception of arithmetic and the role arithmetic has to play after the introduction of numbers in elementary propositions. It is useful here to remember that the *Tractatus* draws a radical separation between the notion of function, which characterizes the sense of a proposition, from the notion of an operation, which is used only as a means of representing a proposition and not as a means of characterizing its sense. The concept of a calculus (and, therefore, of arithmetic) fits into the conceptual framework of the *Tractatus* because of the distance between a truth-function of elementary propositions (and hence a proposition) and its mode of representation. Number, defined as the "exponent of an operation" (6.021), inherits all the characteristics of the notion of operation: a number does not characterize the propositional sense, and occurs only as part of the mode of representation of a proposition. Moreover, numbers occur in propositions not as something intrinsic to their sense, but only because propositions occupy a position in a certain series of propositions.

Now, with numbers entering into the composition of atomic (elementary) propositions the picture has changed. For the first time numbers are taken to characterize the sense of propositions, and there's a need for an explanation of how this occurs, and of how this issue affects Wittgenstein's earlier conception of arithmetic. This would require an elaborate exposition and here I can only mention some telling remarks from MS 105 that support my interpretation. At page 19 of MS 105 Wittgenstein says: "I am apparently thrown back against my will on arithmetic". It seems fairly reasonable to suppose that this unwilling return to arithmetic has to do with the fact that the Tractarian account of arithmetic missed something important. Moreover, Wittgenstein goes on first to mention a strictly Tractarian characterization of number (as a means of representation) and secondly to attempt to understand the occurrence of a number in a proposition as something characteristic of the form of the proposition. He writes:

> The number is a means of representation. When I say: there are 4 books on the table, I could also express the same without the help of the number 4, say, with the help of *another* number. The 4 comes into my representation in that I express it in the form of a proposition about a, b, c, d. (MS 105: 19; Wi1: 7)

That is, the number 4 comes into the representation of the fact that there are 4 books on the table in that the proposition "there are 4 books on the table" is expressed in the form of a proposition about 4 things. Here the number 4 has nothing to do with the position of the proposition in a series, but with the cardinality of a class of things. This in turn explains why, when Wittgenstein decided to give an account of number as something inherent to the sense of a proposition, he was led to discuss the Fregean account of the (cardinal) number and of how numbers were related to concepts and classes. The whole discussion of the nature of number and arithmetic can be found in Chapters X and XI of PB, and will not be treated here.

The second problem concerns the determinateness of sense. In the *Tractatus*, it is said that the requirement that simple signs be possible is the requirement that sense be determinate (3.23). With the abandonment of an analysis in terms of (discrete) simple signs and with the acceptance of extensible signs, Wittgenstein had to think again about how to guarantee the determinateness of sense in the context of this new scenario. He goes into the matter in detail on pages 49-71 of MS 106 (i.e. immediately before the remarks that served as the basis for the discussion of RLF). On page 49 he writes:

> If something is wrong in my foundations, it could only be that elementary propositions do not essentially exist at all, and the analysis yields a *system* of infinitely decomposable propositions. Isn't this *system* sufficient for the requirement of determinateness of sense that I settled? (MS 106: 49; Wi1: 50)

When he wrote this, Wittgenstein was not clear whether it is possible to treat a symbol like φ(2–5) as a logical product of, for instance, φ(2–3) and φ(3–5). He does not know whether the interpolation feature of the expansible sign is really a (potentially infinite) *decomposition*. This turns out to be a decision regarding the possibility of describing a part of a monochrome patch. The possibility that it is implies, as is clear from the last passage cited, the abandonment of the notion of elementary proposition. The possibility that it is not leaves room for a notion of elementary proposition, but recognizes constructions "within" such propositions that are not truth-functional. Wittgenstein seems not to have abandoned the notion of elementary proposition at this moment if only because he says, at the beginning of Chapter IX of PB, that there must be incomplete *elementary* propositions. Moreover, there is evidence both in the manuscripts and in the conversations with the Vienna Circle that Wittgenstein does not eschew the concept of elementary proposition. Indeed at page 35 of MS 108 (dated 24 December 1929) he speaks of a "new conception of elementary propositions" and there is much talk about elementary propositions in Waismann's notes, dated two days before.[12]

If Wittgenstein really left room for the notion of elementary proposition, then he chose the option that it does not makes sense to describe a part of a monochrome patch; the patch is, in some sense, simple. But then again, a description of a patch includes information about its neighborhood to the effect that adjacent patches do not have the same colour. In fact it is probable that this reasoning eventually led Wittgenstein to the recognition that the logical independence of elementary propositions was a requirement that could not be met.

In sum, these two problems arise from the introduction of numbers to represent patches in visual space and are prior to the acknowledgement of logical exclusion between elementary propositions via the colour-exclusion problem. The historical reconstruction of Wittgenstein's thinking in early 1929 indicates that the problem was not crucially responsible for the actual development of his thought.[13]

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[12] See WWK: 38f.

[13] I acknowledge FAPESP for providing financial support during this work. For discussion and criticism I am grateful to Andrew Lugg and three anonymous referees.

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