

INVITED PAPER

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Tree-structured readings of the *Tractatus*

Abstract

I argue that the numbering system of the *Tractatus* lets us see how it was constructed, in two closely related senses of that term. First, it tells us a great deal about the *genesis* of the book, for the numbering system was used to assemble and rearrange a series of drafts, as recorded in MS 104. Second, it helps us understand the *structure* of the published book, as cryptically summarized in the opening footnote. I also discuss an unpublished letter from Anscombe to von Wright from 1948 which contains the very first sketch of a tree-structured reading, and what I believe is Stenius's response to Anscombe's proposal. The paper critically evaluates previous work on tree-structured readings and contends that we need to read the *Tractatus* in both the number order used in the published book and the tree order that Wittgenstein used to draft it. It also considers some of the main ways of turning this complex branching structure into a linear, printed text, and so serves as an introduction to the three tree-structured editions of the *Tractatus* that accompany this paper (the German text, and the translations by Ogden/Ramsey and Pears & McGuinness).

1. The significance of the *Tractatus* numbering system

Wittgenstein's *Tractatus*, first published in 1922, has not only given rise to an enormous, and extraordinarily diverse, philosophical literature, but has also inspired and influenced readers and artists of every kind as a work of art in its own right. Nevertheless, there is an almost complete lack of scholarly agreement about even the most

elementary exegetical matters. The *Tractatus* is a canonical work of early analytic philosophy and a modernist masterpiece, yet there is so little agreement about how to approach it that it can be difficult to know where to start. The book has generated an extraordinarily wide-ranging debate that is approaching its centennial and already has an extraordinarily complex history (see Stern 2003 and Biletzki 2003 for complementary histories of the first eighty years of *Tractatus* interpretation).

However, until very recently, almost all readers of Wittgenstein's *Tractatus Logico-Philosophicus* have taken it for granted that it should be read in the order in which it was printed. In part, that is because it ordinarily goes without saying that a book is to be read from beginning to end. There are, course, well known exceptions to that general rule, such as reference works, or those written by authors who say in the introduction that some readers may wish to skip certain chapters or begin by reading certain key parts. However, *Tractatus* is the Latin word for "treatise", and a logical-philosophical treatise (or a "Logisch-Philosophische Abhandlung", the words Wittgenstein ordinarily used to speak of his book) is just the kind of formal and systematically written book that one would expect to need to read from beginning to end.

The main text of the book consists of over five hundred numbered remarks, starting with 1, ending in 7. All of them are arranged in numerical order; all but the seven whole-numbered remarks make use of from one to five numbers after the decimal point. For instance, the opening remarks are numbered as follows: 1, 1.1, 1.11, 1.12, 1.13, 1.2, 1.21, 2, 2.01, 2.011, 2.012, 2.0121, 2.0122, 2.0123, 2.01231, 2.0124, 2.013, 2.0131, 2.014, 2.0141, 2.02... That intricate numbering system, taken together with the extremely compressed way in which those remarks are written, has led readers to give particular attention to the relationship between beginning, middle, and end, and to a focus on how the various parts of the book are connected.

Wittgenstein's only official explanation of how the decimal numbers are supposed to enable the reader to take in the structure of the book is to be found on the first page of the book, in the form

of footnote number one, attached to remark 1. It consists of just two lapidary sentences:

The decimal numbers assigned to the separate remarks indicate the logical weight of the remarks, the stress laid on them in my exposition. The remarks *n.1*, *n.2*, *n.3*, etc., are comments on remark No. *n*; the propositions *n.m1*, *n.m2*, etc., are comments on the remark No. *n.m*; and so on.

More than a few *Tractatus* interpreters have regarded both the footnote and the numbering system as not only unhelpful but positively misleading. In a review of the book published in the *Philosophical Review* in 1924, Theodor de Laguna contended that the numbering made the book *less* clear and *less* comprehensible, asserting that “to follow the numbers is a constant distraction from sense” and that “the writer himself sometimes get mixed up” (De Laguna 1924, 104; Copi and Beard 1966, 25). However, de Laguna’s assessment of the value of the numbering system, like any other such evaluation, is only worth as much as his account of what is involved in following the numbers, which in his case is disappointingly meagre.

De Laguna’s quick dismissal of the numbering system found supporters among those who wrote the first book-length studies of the *Tractatus* in the late 1950s and early 1960s. Erik Stenius’s *Wittgenstein’s Tractatus*, subtitled “A Critical Exposition of its Main Lines of Thought”, begins with a chapter on the structure of the *Tractatus*. He opens his discussion of “The Formulated Principle of the Numbering” (Stenius 1964, 3) by observing that Wittgenstein is “speaking like a mathematician” in his footnote, “introducing the letter ‘*n*’ to express an arbitrary number” (Stenius 1964, 3), but after a very brief discussion of some questions about how it is supposed to work, Stenius soon concludes that “(thank heaven!) he does not keep consistently to any rule” (Stenius 1964, 4). Max Black’s paragraph on the topic in his voluminous line by line reading of the book, *A Companion to Wittgenstein’s Tractatus* is even more dismissive. There, he summarily asserts that “the device is so misleading here as to suggest a private joke at the reader’s expense” (Black 1964, 2). In one of the first papers to argue in detail for the value and importance of the *Tractatus* numbering system, “Is the Numbering System in

Wittgenstein's *Tractatus* a Joke?" Kevin Gibson accurately summarized this dismissive consensus as follows: "The numbering system characteristically generates one of several reactions: that Wittgenstein started with a system, but he was inconsistent or confused; that the system makes no sense; or that it is so esoteric as to be unfathomable." (1996, 139).

At first sight, this widespread rejection of the idea that the numbering system of the *Tractatus* is of any value as a guide to the book's structure may seem to be in tension with my opening claim that almost all readers have taken for granted that the book should be read in the order in which it was written. But very few of those who have regarded the numbering system as confused or incomprehensible have excluded the idea of reading the book in number order from beginning to end. Their criticism was directed at the idea that the footnote about the numbering system, or some more esoteric way of making use of that decimal numbering, would yield any insight into the book's structure. Such criticism was entirely compatible with taking for granted that the book should be read in number order. Black's *Companion*, for instance, is arranged into ninety chapters, each of which is devoted to a series of number ordered remarks, starting with a chapter on the first seven (from 1 to 1.21) and ending with a chapter on the last three (from 6.53 to 7).

Nevertheless, there can be no doubt that Wittgenstein took the numbering system of *Tractatus* very seriously. When an editor who he had asked to publish it asked Wittgenstein if it could be published without the numbers, he replied in the strongest possible terms that the decimal numbers were absolutely indispensable, and must be printed next to his remarks: without them the book would be an "incomprehensible jumble", for "they alone make the book surveyable and clear" (letter to von Ficker, 6 December 1919, translation from Hacker 2015, 652). "Surveyable" ("übersichtlich", literally, overview-able) is a key term of art for Wittgenstein and carries the sense of making it possible to take in a complex structure at a glance, in the way that one can grasp the lay of the land by looking at a landscape from the top of a well-placed tree or ladder.

Despite Wittgenstein's insistence on the crucial role of the numbering system in the letter I have just quoted, and his outline of

how the system works in the footnote on the very first page of the book itself, surprisingly few interpreters have given those remarks serious consideration. In part, that is because Wittgenstein himself had very little else to say about the numbering system, and that explanatory footnote is so short that it raises many more questions than it answers. As a result, any interpretation of the precise significance of the numbering system has to be a matter of detective work. It involves working out *how* that system was supposed to “make the book surveyable and clear” by means of a close examination of how the author made use of the numbering system. For the first fifty years after the publication of the *Tractatus*, the key to an answer—the author’s use of the numbering system to put the book together, both to assemble a series of successive drafts, and then to rearrange them—remained entirely hidden from view. Indeed, even once the history of the book’s composition became part of the scholarly record, it took a great deal of further work before anyone recognized its full significance.

1a. The construction of the *Tractatus*

In this essay, I argue that once it is properly understood, the numbering system of the *Tractatus* enables the reader to take in how it was constructed, in two closely related senses of that term. First, it provides a great deal of information about the *genesis* of the book, for the numbering system was used to assemble and rearrange it. The numbering system helps us see how the book was put together or assembled from the various parts of which it is composed, especially when it is considered together with the very similar but significantly different numbering used in MS 104, the wartime manuscript volume which records a number of Wittgenstein’s successive drafts of the book.

Second, the use of that numbering system in the book’s genesis helps us understand the *structure* of the published book, as cryptically summarized in the opening footnote. We shall see that the decimal numbering system, and the way in which Wittgenstein used that numbering system to organize his remarks gives the book a quite specific kind of hierarchical structure, namely a logical tree.

In his very first article on the *Tractatus*, published in 1956, Brian McGuinness presciently highlighted a fact about that intricate numbering system that has only recently begun to receive the attention it deserves. The unusual complexity and intricacy of the numbering system is evidence of a great deal of hard work and careful planning on the author's part, and so one should expect that there would be much to be learned from paying closer attention to that system:

That a system of numeration so troublesome for an author to devise will give many useful indications to the interpreter, is a truth that has only to be stated to be acknowledged. (McGuinness 1956, 202; Copi and Beard 1966, 137)

Despite the prominent role that the numbering system plays in the arrangement of the book, and despite many attempts to explain precisely how it works, in practice it took a very long time before readers identified any of the “useful indications” McGuinness anticipated. Indeed, in the conclusion to Gibson's paper on the *Tractatus* numbering system, immediately after recommending that we follow McGuinness's advice in the passage quoted above, Gibson rightly observed that “the truth that the apparently baroque system of numbering may help in understanding the work has not only rarely been stated, it has often been denied” (1996, 147). While McGuinness saw from the start that the very complexity and intricacy of the numbering system ought to be a valuable source of information for an interpreter, it was not until the late 1980s, long after he had edited *Prototractatus* (Wittgenstein 1971), an edition of an early draft of the *Tractatus*, that he began to see *how* Wittgenstein had used the numbering system to write and assemble his book (see McGuinness 1989, 1996, 2002). For it turns out that the key to appreciating why the decimal numbering system mattered as it did to Wittgenstein, is to see the absolutely indispensable role that it played in putting the book together in the first place. In other words, any satisfactory answer to questions about the *structure* of the book will turn on our understanding its role in the book's *genesis*.

However, the first publication to make an extended case that the book should be read in tree order—Verena Mayer's path-breaking paper, “The Numbering System of the *Tractatus*”—was not

published until 1993.¹ Unfortunately, the paper has not received the attention it deserves in recent work on the *Tractatus* numbering system, even though most of the points that she first made there have since become taken for granted. Mayer's crucial contribution to our understanding of the book's structure was not just that she was first person to advocate in print in some detail for reading the book along these lines. It was also that she discovered how the numbering system came about in the first place: she showed, in convincing detail, that Wittgenstein himself had made use of it in order to arrange his work while he was engaged in writing the sentences that make up the book and working out how to arrange them in the right order. Mayer stressed that she was not giving yet another interpretation of the numbering system, but rather "a simple explanation of how it came about. This genetic explanation is conspicuous on reading the *Prototractatus*, but curiously has not been mentioned by the various interpreters of the *Tractatus*."² The core insight that informs Mayer's construal is that "the numbering-system of the *Tractatus* reflects primarily a method of composition."³

Mayer's insight turned on a close examination of the manuscript of Wittgenstein's first draft of *Tractatus*, which was published in facsimile as part of *Prototractatus* (1971). That manuscript, now known either as MS 104 (for its place in von Wright's numbering system) or Bodleianus (because it is now owned by the Bodleian Library in Oxford) is both very similar to the final text of the *Tractatus* and very different. Because the full significance of the order in which the remarks were written down was not yet appreciated, the focus of that book and of von Wright's introductory essay (von Wright 1971), is on the text's relationship to the *Tractatus*, not the composition of MS 104. That focus is already made clear in the wording of the book's subtitle: "an early version of *Tractatus Logico-Philosophicus*". Consequently, the text of the first 103 pages of MS 104 was

¹ Gibson's paper on the numbering system of the *Tractatus* (Gibson 1996), apparently written independently of Mayer's, advocates a reading of the Tractarian numbering system akin to hers, but without her attention to the method of composition.

² Mayer 1993, 110.

³ Mayer 1993, 112. Oddly, Hacker uses this very turn of phrase when he overlooks Mayer's paper and instead attributes this discovery to Bazzocchi's later work, saying that Bazzocchi first had "the insight that the numbering system indicates a logical tree, realized how it is to be read, and *grasped the method of composition*" (Hacker 2015, 652; italics are in the original).

rearranged in the familiar numerical order, while the last fifteen pages of “corrections” were left out, as they belonged to a later stage of revision that could not be fully reconstructed from the available evidence. The numbering used in MS 104 and the overall arrangement that it produces when used to rearrange the remarks into number order is for the most part very similar to the order in the finished book.

The immediate result of this enormous amount of careful and conscientious scholarly work was very disappointing: it was hard for the first generation of readers of the *Prototractatus* to see what, if anything, there was to be gained or learned from this edition.⁴ The edited text looked too much like the familiar text of the *Tractatus* to be instructively different, while the set of photographic facsimiles of each page of the original manuscript that preceded that text seemed quite opaque. In other words, while the published *Prototractatus* looks very similar to the final *Tractatus*, the source manuscript on which that book was based was put together in a very different way. Indeed, while von Wright did not himself provide any further discussion of the “differences in the arrangements of the thoughts” that “are probably the most interesting differences between the two works”, (Wittgenstein 1971, 2) his work, and the work of his co-editors, made those materials available in a form which provoked others to identify those differences. This may well have been one of his most important contributions to our understanding of the complex relationship between MS 104, *Prototractatus* and *Tractatus*.

Roughly speaking, the far-reaching *similarities* between MS 104 and the *Tractatus* can be described as a matter of *content*: one can find an earlier version of almost every remark in the *Tractatus* in the manuscript, and while there are over four hundred places where there are differences in wording, the differences are rarely substantial. There are only a few dozen remarks in the manuscript that do not occur in the *Tractatus* at all, and a similar number of remarks in the *Tractatus* that have no precursor in it. The main *difference* between the manuscript and the *Tractatus* is that the remarks are *arranged* very differently: although there are often short sequences

⁴ For further discussion of the early reception of *Prototractatus*, see Stern 2016, parts 4-5. Part of this discussion is based on that material.

of remarks in the manuscript that occur in the same order in the published book, the manuscript as a whole is arranged in a very different order to the numerical sequence one finds in the finished work. However, there are many more numbered remarks in the manuscript, each consisting of a single short paragraph, often only a sentence long, than there are in the *Tractatus*, as those remarks were often consolidated into a single numbered remark made up of a number of those very short paragraphs. In the revision process that led from the arrangement in the manuscript to the number order published typescript, many such groups of remarks were moved around and renumbered, though the changes were almost always made to the numbers after the decimal point, and the numbering system as a whole was streamlined and simplified: the *Prototractatus* numbering system uses up to nine decimal places, while the *Tractatus* uses only five.

What Mayer noticed is that there is much more to be learned from this preliminary manuscript if one reads it in the order in which it was first written down, instead of rearranging it into the familiar Tractarian number order that was used in the published *Prototractatus*. She observed that

Wittgenstein reviews the ‘scaffolding’ propositions listed on the first page one by one, by complementing, developing and commenting on them. In doing so he adopts, at least in the beginning, a cyclical procedure. Thus by page 8 we already find comments on all propositions of the first page up to and including [*Prototractatus*] 5. Afterwards there is a new round, in which the previous comments are complemented and commented on. By page 22 this process has already lead to very long numbers like [*Prototractatus*] 4.102265. (Mayer 1993, 112)

In other words, from the very beginning Wittgenstein used his numbering to enable him to assemble and organize his remarks. Beginning with the whole-numbered remarks, he first added remarks with a single decimal, and so on. The first surviving numbered page, page three of MS 104, includes drafts of the first six whole-numbered remarks but interpolates several sequences of numbered remarks with just one decimal: 1.1; 2.1, 2.2; 3.1, 3.2; 4.1, 4.2, 4.3, and 4.4, thus adding the first branches to the trunk. Double decimal sub-branches begin on the page after that, which includes such sequences as 2.01,

2.02; 2.11, 2.12, 2.13, 2.15, and 2.16; 2.14 turns up on the next page. Mayer puts this “crucial point” as follows:

an examination of the facsimile edition of the *Prototractatus* reveals that propositions with numbers $n + 1$ by and large succeed propositions with n digits, while at the same time there are often several pages between proposition n and the corresponding comment with $n + 1$ digits. (Mayer 1993, 113)

From each of the first six whole-numbered remarks, numerical sequences branch, starting with one-decimal series such as 1.1, 1.2; from these nodes, further branches stem.

The first hundred or so pages of MS 104 record the addition of further branches, providing us with a detailed log of the process of adding, and sometimes rearranging, progressively finer branches to those main branches he had initially planned out in the opening pages of his manuscript. Throughout the process of construction recorded in MS 104, remarks are added to the tree-structure, not to a numerical sequence, by using the numbering system to indicate where they belonged and were written down in the form of a log of new sequences of remarks. Of the 284 remarks in the first twenty-eight pages of MS 104, only 14 can be traced back to earlier drafts. This material, which Kang (2005, 3) dubbed the “*Core-Prototractatus*” was almost certainly composed in 1915 and takes the form of an outline of most of the main themes of the first five whole-numbered remarks, ending with remark 6. From page 28 onward, further remarks, many of them already drafted and written up in preparatory notebooks, were then attached to this skeletal structure, using the numbering system to do so.

When Wittgenstein began to assemble his book, very likely some time in spring or summer 1915, he was still in the midst of writing it, yet he had clearly arrived at a point where he needed to work out how to arrange the parts that he had already written, many of which had been written down in earlier manuscript notebooks. So it could not simply be written up in the ultimate numerical publication order. If he had not been serving in the Austrian army during the first world war, he would probably have engaged a typist to produce a typescript of the material for his book, and he would then have put them together in the order he thought best. Instead, remarks in the

manuscript were written down as Wittgenstein decided to add them to his book draft, and decimal numbers were used to keep track of the order to be used in the book. Almost all the remarks in the manuscript have a number (of some length or other) from 1 to 7. Crucially, his decimal numbering system was the device that enabled Wittgenstein to organize, review, and repeatedly reorganize his work in progress, and thus compose successive drafts of his book, despite the very limited resources available to him while serving as a front-line soldier.

In October 1915, Wittgenstein wrote to Russell that he had recently done a great deal of work, and that he was

in the process of summarizing it all and writing it down in the form of a treatise [Abhandlung]. ...If I don't survive [the war], get my people to send you all my manuscripts: among them you'll find the final summary [letzte Zusammenfassung] written in pencil on loose sheets of paper. (Wittgenstein 2012, 84-85)

That loose-leaf “final summary” has not survived, but it is likely that it consisted of some kind of a tree-structure arrangement of his book in progress, as a sequentially ordered arrangement would have involved constant and extensive additions to what had already been composed, while inserting material into sheets containing remarks arranged in a tree structure would have been simple. Certainly, it would have been impracticable to take in either the hypertextual structure or the sequential arrangement of the projected treatise by reviewing MS 104, the bound ledger containing a chronological ordered record of his additions to the book draft.

1b. The structure of the Tractatus

Most of those who have reflected more carefully on the wording of Wittgenstein's footnote to the remark number 1 have taken it to tell us only that remarks with fewer decimal numbers are in some sense more important than those with more decimal numbers and that the numbers with no decimals are the most important. They also acknowledge that there is some kind of hierarchical structure connecting any number with another number that is just like it, but has more, or less decimal numbers at the end, so that 3.001, 3.01, 3.02, 3.1, 3.2, among others, are all comments on 3, and 3.201, 3.202,

3.21, 3.22, among others, are all comments on 3.2, for instance. For instance, De Laguna put this point as follows

In order to show the logical interconnection of his propositions, he numbers them in a very elaborate fashion. Thus proposition 5.2523 is the third comment on the second comment on the fifth comment on the second comment on the fifth main proposition. (De Laguna 1924, 104; Copi and Beard 1966, 25)

We can put these fairly widely accepted ideas about the *Tractatus* numbering system in terms that more closely follow the wording of the footnote. In other words, the decimal numbering system is taken to matter because it does the following three things:

- [1] It provides the *numerical order* in which the remarks are to be read (1, 1.1, 1.11, etc.)
- [2] It gives *greater weight to shorter numbered remarks* (e.g., n over $n.1$, $n.1$ over $n.m1$, etc.)
- [3] It indicates which remarks are *comments on others* ($n.1$ on n , $n.m1$ on $n.1$, etc.)

While all three of these principles are true, they are not the whole truth. If we do take [1] + [2] + [3] to be all that Wittgenstein is saying in the footnote, then we arrive at roughly the position Anthony Grayling extracts from it:

The system is one which anyone familiar with business or official reports can quickly grasp; chief points are marked with whole numbers (1, 2, etc.), comments subordinate to those with a single decimal (1.1, 2.1, etc.) and so on in the standard way. The *Tractatus* is rather elaborate structurally, yielding remark-numbers with as many as five decimals, for example 2.02331; but the principle of the arrangement is straightforward, as described. (Grayling 1988, 28)

However, the arrangement is not as simple and straightforward as Grayling, and others like him, have thought. For what Wittgenstein actually has to say about the nature of the commenting relation in the second sentence of his footnote is considerably more intricate, and more interesting, than the bald summary provided in [3]. Wittgenstein does not characterize the relationship of commenting as a binary hierarchical parent-child relation between pairs of remarks, such as n and $n.1$, one of which comments on the other. Instead, he introduces it as a multiple-member parent-children

relationship between a remark at a given level of the numbering system and a sequence of remarks at the next level down that jointly comment on it, such as n and $n.1$, $n.2$, $n.3$, etc.⁵ What Wittgenstein actually says in his footnote about the numbering system is not just that $n.1$ is a comment on n , but rather that the entire series of remarks that start with $n.1$ are comments on n :

The remarks $n.1$, $n.2$, $n.3$, etc., are comments on remark No. n ; the propositions $n.m1$, $n.m2$, etc., are comments on the remark No. $n.m$; and so on.

In this passage, Wittgenstein draws our attention to the *sibling* relations between remarks at the same level on the tree with a common parent, such as $n.1$, $n.2$, $n.3$ etc., and $n.m1$, $n.m2$, etc. It is these *sequences* of sibling remarks that he calls comments on the remark at the next level up.

In other words, the relationship between the remarks created by the numbering system is not reducible to the hierarchical, vertical, parent-child relation, constituting a set of binary relations between pairs of remarks, one commenting on the other. In addition to those two-level relations between parent and child remarks, such as the relationship between $n.m$ and $n.m1$, or between $n.m$ and $n.m2$, the same-level horizontal sibling relations, between $n.m1$, $n.m2$ (and $n.m3...$) are equally important. In fact, as Mayer observes, it is not the case that every remark is a direct comment on a remark with a corresponding decimal with one less digit at the end. Mayer provided the following examples:

A proposition with the number $n.m3$ is not necessarily a remark on proposition $n.m$ when taken by itself. It is often the case that preceding [sic!] propositions $n.m1$ and $n.m2$ must be taken into consideration as well. Thus 4.0311 and 4.0312 *together* constitute a remark on 4.031. Equally, 4.01 to 4.03 form a comment on proposition 4, which is further developed in 4.04 to 4.06.⁶

Thus, in addition to reading his book sequentially, as we would an official report, we also have to pay attention to each *series* of

⁵ The wording of his explanation of the commenting relation in terms of a series of comments ($n.m1$, $n.m2...$ as comments on $n.m$) is not meant to rule out cases such as 1.2 and 1.21, where 1.21 is the only comment on 1.2.

⁶ Mayer 1993, 114-115.

remarks that makes up one of these sibling sequences, *jointly* commenting on the same remark at the next level up. Taking this relationship seriously involves not only reading the 1s of the *Tractatus* in number order, that is, 1, 1.1, 1.11, 1.12, 1.13, 1.2, 1.21, but also in tree order: first considering 1 as part of the sequence of whole numbered remarks that begins with 1 and ends with 7, then 1.1-1.2 as a comment on 1, 1.11-1.13 as a comment on 1.1, and then 1.21 as a comment on 1.2. Likewise, one should not only read the 2s in the *Tractatus* starting at 2 and proceeding in number order, ending at 2.225, but also as a tree-structured family of series of siblings: 2.1-2.2⁷, 2.11-2.19, 2.21-2.22, 2.01-2.06, and so on.

So it would be more accurate to say of the numbering system that:

[3'] It indicates which remarks are *sequences of remarks that comment on others* (*n.1, n.2, n.3, etc., on n; n.m1, n.m2, etc., on n.m, etc.*)

Once we see these sequences of remarks are working in this joint way as going to make up an ordered set of paragraphs that are on the same level and belong together as a comment on a remark at the next level up, then it is a natural next step to read those remarks in that order, even if they are printed on separate pages. So we should note that the numbering system also does the following

[1'] It provides an alternative reading order: arranging the sequences of sibling remarks that comment on their parent remarks as a *tree structure* (1, 2, 3, 4, 5, 6, 7; 1.1, 1.2; 1.11, 1.12, 1.13; ...)

1c. The Iowa Tractatus map

[1] amounts to a statement of an assumption usually regarded so obvious that it was very rarely explicitly articulated, and had seemed

⁷ Throughout this paper, I will use a dash between two numbered remarks as an abbreviation for the sequence of remarks on the same level of the tree-structure that begins with the first and ends with the second. Thus, "1-7" is to be understood as referring to the sequence consisting of 1, 2, 3, 4, 5, 6, 7, the seven remarks at the top level of the tree structure, and not to all 526 remarks in number order, and "1.1-1.2" refers to the sequence consisting of 1.1 and 1.2, and not to the number order series consisting of 1.1, 1.11, 1.12, 1.13, 1.2.

to need no defense, namely that the book should be read *sequentially*, from beginning to end. I will also refer to this method of reading the book as the “number order” method. The new alternative, as stated in [1], is to *read the book as a tree-structure* determined by the author’s numbering system. I will also refer to such methods of reading the book as “tree order” methods.

However, it is very difficult to read the book as a tree-structure while working with the traditional printed text. The seven remarks that make up the “trunk” of the tree-structure are each printed on a different page, and with the exception of the very short set of remarks beginning with “1”, so too are almost all of the remarks that make up the next two levels of “branches”, such as those with one decimal (such as 2.1 and 2.2) and those with two (such as 2.01, 2.02.... 2.06, and 2.11, 2.12, ... 2.19). For most of the twentieth century, those readers who were not blessed with a photographic memory who wanted to carefully contemplate the connections between such sequences had no good alternative to copying out the relevant passages (or cutting up a copy of the book.)

It was not until the 1990s, with the widespread availability of word processing and easy access to the text from Project Gutenberg that it became a relatively straightforward matter to rearrange the text in this way. Jonathan Laventhol (1996) was the first person to put a tree-structured arrangement of the full Ogden/Ramsey translation online.⁸ The starting page shows the seven main propositions in order; clicking on the number “1” takes one to a page showing remark number 1 at the top, and remarks 1.1 and 1.2 below it; clicking on 1.1 leads to a page with 1.1 at the top, and 1.11, 1.12 and 1.13 below it; clicking on 1.2 leads to a page with 1.2 at the top, and 1.21 below it. And so on. A very wide variety of tree-structured *Tractatus* websites have since been developed, presenting the text of the book along the lines suggested by Wittgenstein’s initial footnote.

In 2015, I led a team that developed the [Iowa Tractatus Map](#), an online tool that organized its presentation of a tree-structured text around a subway-style map of that structure.⁹ On the approach taken

⁸ See Laventhol 1996.

⁹ The [Iowa Tractatus Map](#) is a graphic representation of this structure that allows the reader to read the book as a tree structure, making use of the same three texts—tree-structured

there, we start with a horizontal “main line” at the top of the page, the series of whole-numbered remarks, (1, 2...7), each of which is represented by a station on that line. Each series of remarks that comment on one of those top-level remarks is a single vertical line on the map, branching off one of the first six junction stations. One line branches off remark 1 (1.1-1.2); two lines branch off remark 2 (2.01-2.06 and 2.1-2.2); three lines branch off remark 3 (3.1-3.5, 3.01-3.05, and 3.001), and so on. Likewise, each series of remarks that comments on one of those next level remarks branches off horizontally from those remarks, each of which is thus another junction station. And so on.

The use of color is not, strictly speaking, necessary, as the nature of these relations is fully represented by the use of numbers as names for the stations, and the lines that connect them. However, we have found it helpful to follow the convention of using a spectrum of colors to systematically indicate the different kinds of line, starting with purple for the top line, followed by red, orange, green, aqua, and blue for the five decimal levels used in the *Tractatus* numbering system. As remarks containing a single or double zero are not comments on a non-existent remark ending in zero but rather the number preceding the zero—for instance, 4.001 and 4.01 each begin a series of remarks on 4, they are equipollent to the series beginning with 4.1, and so the three are colored in increasingly pale shades of the same red, making the 4.01 line a dark pink, and the 4.001 line a light pink.¹⁰ This makes it easier to keep track of the relationship

editions of the original German, the Ogden/Ramsey translation, and the Pears & McGuinness translation—as the print edition of those materials accompanying this essay (Wittgenstein 2023) published in the “From the archives” section of this volume. For further discussion of the Iowa map, see Stern (2016, 2018, 2019). The Map site also includes a full list of online editions of the *Tractatus*, and links to those web pages.

¹⁰ I take “equipollent” from Peter Hacker (2015) as a convenient way of expressing the idea that such remarks containing a zero (such as 4.01 and 4.001) are equally significant remarks on the remark whose number comes before the zero (in this case, 4) as the corresponding remark without a zero (in this case, 4.1) from Hacker’s discussion of the topic (2015, 652, 655, 659). However, it is worth noting, as Hacker does not, that remarks containing zeros in MS 104 do not always seem to be equally significant to the corresponding remarks without a zero, but rather are just a convenient way of adding a second or third series of comments to a given remark. There are many more remarks containing zeros in MS 104 than in the *Tractatus*. Most of those remarks are moved around and renumbered in the post-MS 104 revision process, so that they no longer use zeros in the final text. This strongly

between the various kinds of lines that make up the map. The *Prototractatus* map is arranged on the same lines; the chief differences between the maps arise out of the fact that the earlier version is more finely divided, containing many shorter, separately numbered remarks that were consolidated into single remarks.

Readers can zoom in on any part of either map, and then move around in it, or zoom out to see the whole. Clicking on any one of the individual numbered stations, each of which stands for a remark in the text, brings up a panel containing the associated text for that remark. Clicking on the lines connecting the stations, each of which stands for a sequence of sibling remarks and the remark that they comment on, brings up a panel containing the text of those remarks. For instance, clicking on the line that includes *n.3* brings up the text of the whole of that branch (e.g., *n.1, n.2, n.3...*), with the text for the junction station, the remark that it comments on, namely *n*, at the top. The default text is the German original, but a dropdown menu in each text panel allows the reader to choose either of the canonical English translations.

In addition to providing a subway-style map of the complete text of the *Prototractatus*, or the first 103 pages of MS 104, our map site also provides parallel access to the earlier stages, or “strata” of composition, contained within the source manuscript for the *Prototractatus*. By choosing different start and end pages at the top of that map, one can look at different stages in the construction of the *Prototractatus*: the chosen pages are in color, the others are greyed out. In this way, one can look at the text of different stages in the construction of the *Prototractatus*, such as Kang’s “*Core-Prototractatus*” (up to page 28), Potter’s “1916 *Tractatus*” (up to page 64), or McGuinness’s “proto-“*Prototractatus*”” (up to page 71), or and map the changing arrangement of the project as it was gradually assembled. However, because the dating of these stages is a matter of scholarly debate, and the site is intended as a resource for interpreters with different approaches, we did not build in any particular hypotheses about the dating. Instead, we simply provide

suggests that one use of zeros in the numbering system in MS 104 seems to have been as a temporary place-holder location. For instance, the great majority of the remarks beginning with 4 and 5 in MS 104 contain a zero, but only a small fraction of those remarks use a zero in their *Tractatus* number.

information about the page on which each remark first appears and leave it to the reader to explore the various layers.

Researchers can consult Schmidt (2016) and Pilch (2016) for facsimiles and transcriptions of many of the key documents, and there is a wealth of information about the structure of MS 104 and its relationship to both the *Tractatus* and *Notebooks 1914-1916* in Geschkowsky (2001). However, all this material is only available in German, and its overall structure is far from easy to take in. Consequently, in collaboration with Joachim Schulte and Katia Saporiti, I have been working on the first complete translation of the *Tractatus* and its German sources,¹¹ with the aim of making it possible for English readers to read them as an interrelated body of work and trace the relations between the different texts.

While working on this retranslation project, it became clear there is also a need for an easily accessible, public-domain tree-structured edition of the *Tractatus* for readers who simply want to read the original German, or the two best-known translations, in tree order. Wittgenstein's own description of the tree structure in his footnote to the first remark of his book leaves open a remarkably wide variety way of possible tree-structured arrangements, which is why the title of this essay refers to "tree-structure readings" of the *Tractatus*. Let us turn, then, to a consideration of some of the main ways of turning this complex branching structure into a linear, printed text.

2. Tree-structured editions of the *Tractatus*

Consider the case of the first seven numbered remarks in the familiar number order edition, that is, those beginning with a "1". As they take up less than a page of printed text, it is possible to take in their tree-structured arrangement simply by looking at the first page of the number ordered text and attending to those tree order sequences of remarks in that order. In other words, one can first look at 1.1 and 1.2 as a single unit, expanding on what is said in 1, and then attend

¹¹ MSS 101-104, in von Wright's catalogue. While almost all of this material is now available in translation, the translations by Pears and McGuinness (of the first 103 pages of MS 104, in Wittgenstein 1971), Anscombe (of the recto pages of MSS 101-103, in Wittgenstein 1961) and Perloff (of the verso pages of MSS 101-103, in Wittgenstein 2022) each make very different choices about how to select, translate, and present the text.

to the sequential series consisting of 1.11, 1.12, 1.13 as a comment on 1.1, and then look at 1.21 as a comment on 1.3. Daniel Kolak's translation of the *Tractatus* introduced a way of highlighting this "numerical topography" (Kolak 1998, ix) while retaining the number ordered presentation of the text: the primary level remarks (1-7) are left unindented, the secondary level remarks (comments on the whole numbered remarks) are indented once, the tertiary level remarks (comments on the secondary level remarks) are indented twice, and so on. In this way, the blocks of text at different levels that I have just listed will immediately draw the reader's attention, without having to first look carefully at the numbering of each remark in order to distinguish and attend to the various levels.

Following Kolak's model, the 1s can thus be simultaneously presented in both number order and tree order:

1 The world is all that is the case.¹²

1.1 The world is the totality of facts, not of things.

1.11 The world is determined by the facts, and by their being *all* the facts.

1.12 For the totality of facts determines what is the case, and also whatever is not the case.

1.13 The facts in logical space are the world.

1.2 The world divides into facts.

1.21 Something can either be the case or not be the case while everything else remains the same.

¹² Here, and throughout this essay, all otherwise unattributed quotations from the *Tractatus* are taken from the draft of the translation of the *Tractatus* that I am working on with Joachim Schulte and Katia Saporiti, forthcoming from Cambridge University Press. Readers who wish to consult the German text of this (and any other) tree-structured arrangement, or the Ogden & Ramsey or Pears & McGuinness translation as a tree structure will find them in the accompanying tree-structured edition of those texts in this journal. The texts are published in the From the Archives section of the present Volume, DOI xxx.xxx.xxx.

This approach to displaying a tree ordered arrangement works well for those sequences of remarks that are short enough that they can be taken in at a glance. So I have followed it in my tree ordered edition for those series of remarks that, like this first series, can fit onto a single page, or a single pair of facing pages.

However, this way of using indentation is only helpful when one is dealing with such a small group of remarks. It cannot solve the principal problem with trying to follow a tree-structured reading while looking at a number ordered text, which is that the great majority of primary, secondary, and tertiary level sequences of tree ordered remarks are spread over many pages of a number ordered text. The primary level, namely the series of seven whole-numbered remarks starts on the first page and ends on the last; apart from 1 and 2, each of these remarks is separated from the next one by so many others that it is no easy matter to find the next one, and impossible to keep them all in view at the same time. The same difficulty arises when one tries to read most of the secondary and tertiary level remarks in tree order, even when making use of Kolak's indentations: in almost every case, it is impossible to read through the whole of any one tree ordered sequence without continually turning pages, and so one cannot look over at the entire series as a whole.

The natural solution to this problem, first adopted in Jonathan Laventhol's online hypertext edition of the *Tractatus* (Laventhol 1996) and followed in Luciano Bazzocchi's printed tree-structured editions, is to replace a number order presentation with an arrangement that groups remarks in tree order. In Bazzocchi's *The Tractatus According to Its Own Form* (Bazzocchi 2014a), his edition of a revised version of the Ogden/Ramsey translation, Bazzocchi devoted a separate page to each branch of the tree structure, preceded by the remark that it comments on. In his later edition of a revised version of the Pears & McGuinness translation (Bazzocchi 2021), he followed the same general principle, but used a blank space to indicate the break between each unit, instead of a page break. Following this approach, the remarks that make up the 1s can be presented along the following lines, using underlining to indicate remarks that have lower-level comments further on in the text:

1 The world is all that is the case.

1.1 The world is the totality of facts, not of things.

1.2 The world divides into facts.

1.1 The world is the totality of facts, not of things.

1.11 The world is determined by the facts, and by their being *all* the facts.

1.12 For the totality of facts determines what is the case, and also whatever is not the case.

1.13 The facts in logical space are the world.

1.2 The world divides into facts.

1.21 Something can either be the case or not be the case while everything else remains the same.

As I have already proposed, I believe that a hybrid approach that makes use of each method for those cases to which it is best suited is the best way of presenting a tree-structure as a linear text, and so it is the one I have adopted in the tree ordered arrangements of the text that accompany this essay. One can take advantage of Kolak's method of indenting several different tree ordered levels within a number order series of remarks when the series is short enough to fit on a page or two, which results in a more concise and compact presentation. In such cases, there is no need for the repetition of remarks that are both parents and children. For instance, in the tree-structured rearrangement immediately above, 1.1 and 1.2 have to be printed twice: first in their role as children of 1, and secondly in their role as parents, of 1.11-1.13 and 1.21, respectively. On the other hand, when a longer series of remarks and its successive layers of comments cannot fit into a single pair of pages, the

Laventhol/Bazzocchi method of rearranging remarks into a tree ordered text is the best way of handling such cases.

Indeed, once we move beyond the 1s and consider the much larger and more complex tree-structures of repeatedly branching series of remarks that make up the 2s, and the even larger and more complicated structures of the 3s, 4s, 5s and 6s, there is no good alternative to presenting the higher level series of remarks, such as 2.1-2.2, separately from the many remarks at lower levels of the numbering system that start with “2.1” and “2.2”. For instance, in the case of the 2s, my edition presents what one might call the “principal branches” of that part of the book—those tree-structured series of remarks that are, in turn, commented on by a number of other lower-level remarks—namely, 2.01-2.06, 2.011-2.014 (page 3)¹³, 2.021-2.027 (page 6), 2.1-2.2, and 2.11-2.19 (page 9), in tree order. We need to read the 2s in the *Tractatus* as starting at 2 and ending at 2.225, but also as a tree-structured family of series of siblings: 2.1-2.2, 2.11-2.19, 2.21-2.22, 2.01-2.06, and so on.

However, when we reach the lower-level series of remarks that comment on those principal remarks, those sets of remarks can fit onto a single page or at most two, and so the relationship between different levels there can be presented much more compactly and simply using indentation. For instance, the set of remarks on 2.15 involve three different levels of decimal numbering (2.151 comments on 2.15, 2.1511 comments on 2.151, and 2.15121 comments on 2.1512), and so needs to be broken up into three separate logical units if one follows the Laventhol/Bazzocchi model, but can be presented as a single series of remarks on a single page if one uses indentation to indicate those relationships (see Wittgenstein 2023, p. 10).

A particular advantage of this hybrid approach is that it draws the reader’s attention to those upper-level series of remarks—at the primary, secondary and tertiary levels—that are indiscernable in a number order arrangement, while retaining the familiar number order arrangement for those cases where one can easily use indents to identify different levels of the tree order. It is these upper-level

¹³ The page references refer to the page numbering in the tree-structured edition published in this volume (Wittgenstein 2023).

tree-structured series that are all but invisible to a number order reader.

First and foremost among these tree-structured connections that are hidden from view to those who only read the book in number order are the links between the seven whole-numbered remarks, the trunk of the *Tractatus* tree itself. Although it will not be possible within the scope of this paper to do more than suggest some of the ways one might go about reading the main branches of the *Tractatus* tree, we do at the very least need to briefly consider what can be learned from examining the trunk of that tree, or in reading those seven whole-numbered remarks one after another. In particular, I want to approach this key text, the opening page of any tree-structured edition, as an exemplary illustration of both the value and the limitations of tree-order readings. While tree-ordered ways of reading the text of the *Tractatus* are often illuminating, it is wishful thinking to imagine that they can provide the basis for a resolution of the principal exegetical questions about the early Wittgenstein that have divided his interpreters and expositors. In particular, Bazzocchi (2010a, 2014, 2021) and Hacker (2015, 2021) have contended that their tree-structure construal of the numbering system decisively supports a traditional reading of the book as systematically clarifying the logic of our language, and equally decisively refutes any reading of it as a ladder that must be recognized as nonsense once one has climbed it. It is certainly possible to read the book along the lines they propose, and it is not my aim here to prove that their reading is entirely mistaken. However, on balance it seems clear that a tree order reading actually strongly supports, rather than undermines, the interpretive approach that Bazzocchi and Hacker oppose.

In our translation, the trunk of the *Tractatus* tree reads as follows:

- 1 The world is all that is the case.
- 2 What is the case—a fact—is the obtaining of states of affairs.
- 3 A logical picture of facts is a thought.
- 4 A thought is a sentence that makes sense.
- 5 A sentence is a truth-function of elementary sentences.
(An elementary sentence is a truth-function of itself.)
- 6 The general form of a truth-function is: $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$. This is the general form of a sentence.
- 7 We must keep silent about what we cannot speak of.

It is striking when one looks at them in this way that the first six remarks form a chain or ladder: a key expression in each of the first five is repeated in the next one in the sequence, and so provides a connection between them. The links making up this chain are as follows: “is the case” (“der Fall ist”) in 1 and 2; “fact” (“Tatsache”) in 2 and 3; “thought” (“Gedanke”) in 3 and 4; “sentence” (“Satz”) in 4 and 5; and “truth-function” (“Wahrheitsfunktion”) in 5 and 6. Putting the first six remarks together, we get a birds-eye view of the main themes of the treatise: it concerns a world of facts, made up of obtaining states of affairs, facts that we grasp by way of logical pictures, in thoughts that are sentences that make sense, and made up of sentences that are truth-functions of elementary sentences. As Mauro Engelmann puts it in his discussion of the “ladder lessons” of the *Tractatus*, each remark “from 1 to 6 presents two major formal concepts that work like vertical side rails that sustain and connect the rungs by means of informal definitions”.¹⁴ In a similar spirit, Alois Pichler observes that “the rungs of the ladder are linked together by an unbroken chain that is built out of the central terms of the cardinal propositions”.¹⁵

¹⁴ Engelmann 2021, 39.

¹⁵ Pichler, forthcoming.

Hacker takes the seven whole-numbered remarks, the trunk of the Tractarian tree, so to speak, to turn on a line of argument that unites them into a single “single sequence of ideas of the utmost generality”:

They are all elegantly linked to each other: 2 proceeds from 1: the world is all that is the case, and what is the case is a fact (which is the obtaining of states of affairs). 3 proceeds from 2: *a thought* is a logical picture of facts, and 4 proceeds from 3: *a thought* is a *proposition* with a sense. 5 proceeds smoothly from 4: *a proposition* is a *truth-function* of elementary propositions, and 6 gives us the general form of *a truth-function*, which is also *the general propositional form*. Finally, 7 follows on 6: anything that cannot be described by means of sentences that are the result of truth-functional operations of successive negation on (bipolar) elementary propositions cannot be spoken of. In particular, nothing that is ‘higher’ can be expressed by well-formed propositions.¹⁶

While this does summarize the gist of what these seven principal remarks have to say, Hacker’s way of relating them misses an important point about how they do the work they do. The first six whole-numbered remarks can indeed be read as a summary of a philosophical theory about the nature of the proposition and what can be said, one that is elaborated in the decimal-numbered branches attached to them. But those very remarks are not, to use Hacker’s own words in his summary of remark number seven, “the result of truth-functional operations of successive negation on (bipolar) elementary propositions” and so it follows they themselves, “cannot be spoken of”. If we see the first six remarks as a ladder, a series of connected steps that lead up to the general propositional form, the culmination and terminus of the book that Wittgenstein planned in the opening pages of MS 104, then the seventh remark is not the final rung, but rather a step beyond it, a matter of throwing away the ladder once one has climbed up it. While there is a great deal of argument and clarificatory elucidation in the *Tractatus*, we are also told in 6.54 that

My sentences are elucidatory in the following way: to understand me you will ultimately recognize my sentences—once you have used them

¹⁶ Hacker 2015, 657.

as a way of ascending beyond them—as nonsense. (You must, so to speak, throw away the ladder after you have climbed up it.)

You must overcome these sentences, and then you will see the world rightly.

A central question for anyone reading this remark is how we are to relate the apparently confident assertions of the book's whole-numbered remarks and this remark's claim if the reader understands the author, then the reader will see that the very words of the book we have just read are themselves nonsensical, a ladder that must be thrown away once we've climbed up it. Put very briefly, I believe the key to answering this question and so understanding the relationship between the apparently constructive project summarized in 1-6 and the apparently destructive overcoming of that project outlined in 6.54 and 7 turns on Wittgenstein's changing conception of his book during the years he was engaged in writing it.

As the book was gradually assembled in its current order over the course of several years, tracing this trajectory can help us to uncover a number of earlier, rather different, versions, and visions, of the book as a whole. The first page of MS 104 consisted of six lines, early versions of the first six whole-numbered remarks, beginning with what we now know as remark number 1, "The world is all that is the case" and ends with an early version of remark number 6, "The general form of a truth-function is:". ¹⁷ Wittgenstein began writing his book convinced that his "whole task" consisted of "explaining the nature of the sentence... giving the nature of all facts, whose picture the sentence is." ¹⁸ The book he envisaged when he wrote that diary entry in January 1915, a book beginning with remark 1 and ending with remark 6, finds its fullest expression in the first sixty or so pages of MS 104, the "1916 *Tractatus*". That first half of his manuscript, almost certainly assembled during the second half of

¹⁷ This is the full wording of the initial draft of remark number 6 on the first page: no formula follows the colon. Note that the formula that follows the colon in the next version, "[$N(\bar{p}_0), \bar{\alpha}, N(\bar{\alpha})$]", in a subsequent draft on page 3 is not the one he ultimately settled on. Remark numbering was not used on that first page, which was later torn out and has been reconstructed by Martin Pilch on the basis of close examination of traces left on the next page (Pilch 2015); numbering begins on the next page.

¹⁸ Wittgenstein 1979, p. 39. 22 January 1915. My translation. Cf. Engelmann's discussion of this passage (2021, 28, 40.)

1915 and the first half of 1916, records the development of an early version of the book, one that built up to the concluding statement of the general form of a truth-function. It contains most of what we now know as the decimal-numbered remarks beginning with 1, 2, 3, 4, and 5, but none of those that follow 6.¹⁹ This book draft, dubbed the “Proto-*Protractatus*” by McGuinness and the “1916 *Tractatus*” by Potter²⁰, contains neither the ultimate conclusion, remark number 7, “We must keep silent about what we cannot speak of” nor those remarks on mathematics, probability, causality, ethics, and religion that would make up the 6s, which are added subsequently. As Michael Potter observes, “What stands out straightaway is that the outline ends not with the injunction to silence of the final published version but with a technical claim... about the expressive power of a certain notation” (2020, 319). However, the book Wittgenstein completed and ultimately published does not end at the top of the ladder with remark 6, but rather with the discussion in the 6s of a whole range of further topics and concludes with remark 7. In other words, I propose that we understand the relationship between 1-6, and the remarks following 6, up to and including remark 7 as a result of the book’s genesis.

Whether or not looking at the *Tractatus* as tree ultimately settles the methodological debates that divide Wittgenstein interpreters, it offers *any* reader of the *Tractatus* an excellent way of reading and approaching the book, one that is particularly accessible and attractive. It is not only an important resource for the expert interpreter, but also to the beginner looking for an accessible way of seeing the connections between the various parts. The book was originally written as a logical tree, and when read in that way, is often much easier to follow. It is often much easier to get a sense of the connections between the various parts of the book if one makes use

¹⁹ For further discussion of the details and of their philosophical implications, see Bazzocchi 2005, 2007, 2010, Kremer 1997, Potter 2013 and 2020 (Part III is devoted to the “1916 *Tractatus*”, the first 64 pages of MS 104, at which point the book would still have ended with remark 6), and Stern 2018, 246-252.

²⁰ Strictly speaking, McGuinness and Potter demarcate this version of the book that does not go beyond remark number 6 slightly differently: Potter takes it to come to an end on page 64 of MS 104, where Wittgenstein introduces the first remark that comes after 6 (6.1); McGuinness draws the line at the point at which remark number 7 is first written down, on page 71.

of the tree-structured arrangement, rather than the number order arrangement in the published *Tractatus*. This approach to the text is particularly helpful for first time readers; I have found that students have a much better sense of how the various parts of *Tractatus* hang together when they read it in this way. Furthermore, the numbering system lets the reader see two crucial ways in which the book was put together or assembled from the various parts of which it is composed: the *structure* of the published book, as cryptically summarized in the opening footnote, and the *genesis* of the book, as it is the numbering system that was used to assemble and rearrange it.

Luciano Bazzocchi's centenary tree-structured edition of the *Tractatus* begins by setting out his rationale for what he regards as the only right way of connecting its numbered remarks, the "precise connections, ...the exact formal hierarchy that has been handed down to us" (2021, 8). The resulting text is elegantly and simply arranged, consistently following a strictly hierarchical principle. Apart from the first page, which consists of the seven whole-numbered remarks, every logical unit, separated from the previous unit and the next unit by a blank space, begins with a single remark, say *n.m*, and is followed by each remark which is a comment on *n.m*, such as *n.m1*, *n.m2*, *n.m3*, and so on. However, Bazzocchi provides very little support for the claim that the precise structure he proposes is the one that Wittgenstein intended. Although Bazzocchi confidently asserts that "we know that Wittgenstein considered the various decimal layers of his numbering as if they were parenthetical inclusions, inside a nested system of brackets within brackets", he offers no direct evidence that Wittgenstein ever said or wrote such a thing. Instead, he takes it to follow from a general principle that Wittgenstein supposedly followed in the process of revision that led to the final arrangement. The argument goes as follows:

Nearly all statements in brackets in the *Tractatus* derive from a contraction of the structure of the manuscript, where they were distinct comments on a further decimal level. For instance, the parenthesis occurring in proposition 5 was originally conceived, on p. 11 of the manuscript, as separate remark 5.01. Thus, we know for sure that Wittgenstein considered parentheses and decimal sublevels as equivalent syntactical signs. (Bazzocchi 2021, p. 13, n.1.)

There are a dozen other places in the *Tractatus* where a statement in brackets originates from such “a contraction of the structure of the manuscript”, namely a part of a *Tractatus* remark in brackets having its origin in a pair of MS 104 remarks, one of which was a comment on the other.²¹ However, there are by my count at least 19 instances of statements in brackets in the *Tractatus* that are clear counter-examples to this general claim. It turns out that this pattern, far from being almost exceptionless, is actually less common than the exceptions to the principle proposed by Bazzocchi. In many cases, a parenthetical statement in a *Tractatus* remark was part of a series of MS 104 remarks at the same level (e.g. *Tractatus* 2.0121 (3)); in others, parentheses were added to a sentence within a paragraph without any change in decimal sublevel (e.g. *Tractatus* 2.0123); in some cases, the brackets were already present in the manuscript (e.g. *Tractatus* 4.04 (2)).²² So we have not been given any reason to think

²¹ They are: TLP 2.0131 (1b) (from PT 2.0141 & 2.01411), TLP 2.06 (2) (from PT 2.06 & 2.0601), TLP 3.14 (3) (from PT 3.213 & 3.2131), TLP 3.143 (3) (from PT 3.162 & PT 3.1621), TLP 3.325 (2) (from PT 3.2015 & 3.20151), TLP 4.003 (2) (from PT 4.0016 & 4.00161), TLP 4.014 (3) (from PT 4.01141 & 4.011411), TLP 4.061 (2) (from PT 4.091 & 4.0911), TLP 4.1272 (10) (from PT 4.1022728 & 4.10227281), TLP 4.431 (3) (from PT 4.421, 4.422 & 4.4221), TLP 4.52 b (from PT 4.43014 & 4.4301401), TLP 4.461 (4) (from PT 4.448 & 4.44801), TLP 4.464 (2) (from PT 4.4486 & 4.44861), and of course Bazzocchi’s own example, TLP 5 (2) (from PT 5 & 5.01).

The number and letter in parentheses after the TLP remark numbers count paragraphs and sentences. For instance, “TLP 2.0131 (1b) (from PT 2.0141 & 2.01411)” indicates that the parentheses in question enclose the second sentence of the first paragraph of TLP 2.0131, that the sentence in question was previously PT 2.01411, and that the first sentence of the first paragraph of TLP 2.0131 was previously 2.0141.

It is not as simple as it might seem at first sight to give a precise count. There are two edge cases that strike me as debatable which I have left out of this pair of lists (TLP 3.323 and 4.122) and others might well count somewhat differently. But what matters is that there are fewer instances of to Bazzocchi’s “parenthetical inclusion” principle that “statements in brackets in the *Tractatus* derive from a contraction of the structure of the manuscript, where they were distinct comments on a further decimal level” than there are counter-examples.

²² The following are instances of unbracketed text followed by bracketed material in TLP originating from a pair of remarks on the same level in PT: TLP 2.0121 (3) (from PT 2.01201 & 2.01202), TLP 2.0122b (from PT 2.0121 & 2.0122), TLP 3.328 (2) (from PT 3.2521 & 3.2522), TLP 3.3441 (2) (from PT 3.2511 & 3.2512), TLP 4.025 (2) (from PT 4.0261 & 4.0262), TLP 4.04 (2) (from PT 4.073 & 4.074), TLP 4.111 (2) (from PT 4.10012 & 4.10013), TLP 4.126 (2) (from PT 4.102253 & 4.102254), TLP 4.1274 (2) (from PT 4.10227253 & 4.10227254), TLP 4.241 (3) (from PT 4.22121 & 4.22122), TLP 4.442 (4) (from PT 4.441 & 4.442), TLP 4.463 (2) (from PT 4.4321 & 4.4322).

The following instances of unbracketed text followed by bracketed material in TLP are based on remarks in PT in other ways than the ones listed above: TLP 2.0123 (2) (from PT

that “Wittgenstein considered parentheses and decimal sublevels as equivalent syntactical signs”, let alone that there is indeed “one right way” of connecting its numbered remarks, or that Wittgenstein handed down to us any such “exact formal hierarchy”.

However, there are many ways of reading a logical tree, or of presenting its contents in linear order, and the ones I have outlined so far are only a few of the possibilities. Rather than insisting that there is only one right way to “comply strictly with the prescriptions afforded by the decimal codes of the *Tractatus*” (Bazzocchi 2021, 8), we would do better to explore the variety of possible readings opened up by what we now know about the tree-structured genesis of the *Tractatus*. In the next section of this paper, I turn to some other ways of reading the *Tractatus* in tree order.

3. A brief history of the first tree-structured readings of the *Tractatus*

In an unpublished letter that Elizabeth Anscombe sent to G. H. von Wright in May 1948, Anscombe suggested reading the book by following successive levels of the numbering system.²³ She described this method of reading the *Tractatus* as follows:

By the way, it occurred to me to try a method of reading it which is pretty obvious but has not been tried by anyone I mentioned it to, and which I think helps: it is to read it in successive steps, first whole

2.0124, brackets introduced into a single PT remark), TLP 2.182 (from PT 2.182, brackets already present in PT remark), TLP 3.144 (2) (from PT 3.1604 & 3.2011), TLP 3.24 (3) (from PT 3.20106, brackets introduced into a single PT remark), TLP 3.42 (3a & 3b) (from PT 3.2104, 3.2141 & 3.2142), TLP 4.024 (2) (from PT 4.025, brackets introduced into a single PT remark), TLP 4.122 (2) (from PT 4.10222 & 4.102231), TLP 4.123 (2) (from PT 4.1022331, brackets already present in PT remark), TLP 4.466 (3) (from PT 4.4491, brackets already present in a single PT remark).

²³ Anscombe’s letter does not include a date. However, on the first page she expressed her support for von Wright’s candidacy for the Cambridge philosophy chair that Wittgenstein had held and congratulates him on his having just been elected in a postscript. As the election took place on May 14, 1948, it seems very likely that she began to write the letter not long beforehand and sent it shortly after. I first became aware of the letter thanks to Thomas Wallgren and Bernt Österman, who have been reading the correspondence between the Wittgenstein trustees as part of a long-term research project on Anscombe, von Wright and Rhees’s relationship to Wittgenstein and the three literary heirs’ contribution to “the creation of Wittgenstein” (Wallgren, 2023).

numbers, then these together with the first decimal point, then up to the second point, and so on.²⁴

Anscombe's suggestion amounted to the first formulation, or perhaps anticipation, of one way of reading the *Tractatus* along tree-structured lines. On the usual way of reading the book in number order, one would start with 1, then move on to 1.1, 1.11, 1.12, 1.13, 1.2, 1.21, 2, 2.01, 2.011, 2.012, and so on. On Anscombe's tree-structured "method of reading", one would start with the primary level, namely 1, 2, 3, 4, 5, 6 and 7, then move on to reading each of those remarks together with the secondary level remarks attached to them, the "first decimal point", namely 1.1-1.2²⁵; 2.1-2.2; 3.1-3.5; 4.1-4.5; 5.1-5.6; 6.1-6.5.²⁶ Her very brief indication of the next steps is not completely specific, but it seems plausible that the next stage would be to repeat the reading of the first and secondary level remarks, but this time including the "second [decimal point]" tertiary level remarks at the appropriate points in the tree structure, beginning with 1.11-1.13 after 1.1.

The method that Anscombe describes is indeed "pretty obvious", in the sense that once one tries it, it rapidly becomes obvious that the alternate reading order is not only extremely helpful, but so well-organized that this way of arranging the numbered remarks must have been carefully planned by the author. Anyone who follows her advice will find, as Hacker puts it, "how illuminating it is to read them *as* sequences".²⁷ Each of the "successive steps" that she describes either takes the form of a table of contents, briefly summarizing the topics that will be discussed in greater detail in the

²⁴ © M C Gormally & R Morgan, reproduced by permission of the literary executors.

²⁵ As previously explained, a dash between two numbered remarks serves as an abbreviation for the sequence of remarks on the same level of the tree-structure that begins with the first and ends with the second. So 1.1-1.2 just means 1.1 followed by 1.2, while 3.1-3.5 means the series consisting of 3.1, 3.2, 3.3, 3.4 and 3.5.

²⁶ Anscombe's first step, the trunk of the *Tractatus* tree, can of course be found on the first page of all three of the accompanying tree-order editions of the book. The second step can be found on the following pages of those editions: 2 (1.1-1.2), 9 (2.1-2.2); 14 (3.1-3.5); 36 (4.1-4.5); 57 (5.1-5.6); 91 (6.1-6.5). Anyone who would like to continue reading further steps in this arrangement of the text without having to leaf through my tree-order arrangement to find the right pages can consult Kevin Klement's "hierarchical" edition of the text, available for download from his *Tractatus* site (Wittgenstein 2022a).

²⁷ Hacker 2015, 657. For some further discussion of "the illumination cast" by 2.11-2.19 and 4.1-4.5, see Bazzocchi 2014a, v-vii, Hacker 2015, 658-670.

smaller branches attached to the remarks that make up that main branch (such as 2.1-2.2, or 6.1-6.5), or a continuous train of thought elaborating on one of the central themes of the book (such as 2.11-2.19, or 6.31-6.37) or in some cases, both at once (1-7, 4.1-4.5). Developing a detailed reading of these passages along these lines would take more time and space than is possible in this essay. However, it should be clear to anyone who reads them that the first two levels of Anscombe's way of reading the *Tractatus* do give us a striking summary of many of the central themes of the book, while also providing a set of topics that are taken up in much more detail in the rest of the book.

In another sense, the method of reading the *Tractatus* that Anscombe sketches in her letter was not at all obvious. Anscombe and von Wright had both been part of the small circle of Wittgenstein's most trusted students for some time before she wrote her letter to him suggesting this method, and not long after they would both become his literary executors. The book had been in print for a quarter century and was already widely recognized as a modern classic. Nevertheless, Anscombe hadn't found anyone who had tried her step-by-step method of reading it, and so thought it worth mentioning, if only parenthetically, to von Wright.

While Anscombe does not explicitly say that she came up with the method of reading she outlined on her own, she would surely have mentioned it if Wittgenstein had suggested it, or something similar, had he done so. It is also worth noting that there is no record of Wittgenstein's ever having explicitly recommended a tree-structured reading to any of the many people who discussed the *Tractatus* with him.²⁸ If Wittgenstein really had thought of it as a key to understanding the book it seems likely he would have recommended making use of it in some of those discussions, and that at least one of those people would have made a note of it. This lends support to the view that as far as the author was concerned, it

²⁸ Nor is there any mention of his having done so in Klagge 2022, a guide to the background to the *Tractatus* that gathers together a very wide range of evidence about what Wittgenstein had to say about the *Tractatus* after it was written. Klagge does not address this question directly but does observe that "Wittgenstein apparently did not emphasize the tree structure to Russell and to Ramsey when he had extensive discussions about the *Tractatus* in 1919 and 1923" (2022, p. 41, n. 17).

played the role of a method of composition, akin to a temporary scaffolding used to support a building while it was under construction, rather than the master key to its final structure.

For the next forty years, few readers gave the numbering system serious attention, and hardly any of them considered the possibility of making use of it along the lines Anscombe had proposed. With hindsight, it ought to be clear to anyone who reads it in the order Anscombe proposed, that Wittgenstein very carefully arranged each series of remarks that makes up each branch of the tree structure so that each of those series of remarks forms a carefully composed sequence of remarks on the remark next up in the tree order. Despite this, as we have seen, it was not until the 1990s, forty-five years after Anscombe wrote her letter, that anyone tried to make the case in print for reading the book along tree-structured lines.²⁹

At first, it may seem odd that Anscombe's path-breaking *An Introduction to Wittgenstein's Tractatus*, the first monograph dedicated to an exposition of the main argument of the *Tractatus*, never explicitly mentions this method of reading at all. Anscombe does take a first step in that direction towards the end of her introductory chapter: she quotes all seven whole-numbered propositions in order, describing them as "the main propositions" (1959, 18). However, instead of giving any attention to the close links between the individual remarks at that level, exploring the way in which key terms are repeated in remarks 1 and 2, 2 and 3, 3 and 4, 4 and 5, and 5 and 6, or discussing their relationship to the sequences of remarks at the next level that are comments on those remarks, she immediately goes on to assert that "the principal theme of the book is the connection between language, or thought, and reality. The main thesis about this is that sentences, or their mental counterparts, are pictures of facts." (1959, 19).³⁰

The theme that Anscombe identifies as central is one that she explores quite independently of any guidance provided by a number order, or a tree order, reading of the text, as she explicitly

²⁹ Stenius's consideration of the top branches of the tree order (1964, ch. 1), discussed below, may count as an exception to this rule, although for the most part his discussion of a tree-structured reading is closer to critique than to advocacy.

³⁰ Anscombe's outline of her way of reading the *Tractatus* is strikingly similar to Wittgenstein's statement of his "whole task" in January 1915, quoted above on page 26.

acknowledged later on the very same page. So in all likelihood, Anscombe did not mention the method of reading the book that she had recommended to von Wright because, as she put it, her book does “not follow... the arrangement of the *Tractatus* at all. That, I think, is something to do when one reads the book for enjoyment *after* one has come to understand its main ideas.” (1959, 19). It is not apparent precisely which “arrangement of the *Tractatus*” she had in mind when she wrote those words, but it seems clear that she would have applied that observation to not only the usual number order reading, but also a tree-structured one. Instead, her book identifies a line of argument that concentrates on the issues that she considered most important. One can see just how different her own way of ordering her discussion is from that found in the book if one considers which chapters of her own book she cites in a series of footnotes attached to her quotation of the seven whole-numbered remarks: chapters 1 and 4 on remark 2; chapter 4 on remark 3; the glossary, chapters 1 and 3 on remark 5; chapter 10 on remark 6, and chapters 5 and 13 on remark 7.³¹

On the other hand, Anscombe did not give up the idea of making use of the reading order that she had proposed to von Wright in 1948. Over forty years later, when she was reviewing and extensively revising a translation of the *Tractatus*, she made use of it in a comment on a draft of a translation of 4.02. 4.02 consists of a single sentence. In Anscombe’s translation, it reads: “This we can see from our understanding the sense of a sentential sign, without its having been explained to us.”³² But what are we supposed to see from our being able to understand a sentence without needing further help? In other words, what does the opening “this” point back to? Reading the text in number order, one would look back to the words immediately before it on the page, in 4.016, but they are no help at all in making sense of 4.02. Anscombe answers this question in a note on the otherwise blank facing page: “‘this’ -- ref. back to 4.01. Might not a footnote be a good idea, indicating one has to take the

³¹ Anscombe 1959, 19, notes 1-6. There are no such references for remarks 1 and 4.

³² Anscombe Archive, Box 5, File 186, p. 120. The Collegium Institute Anscombe Archive at the University of Pennsylvania, The Kislak Center for Special Collections, Rare Books and Manuscripts. © M C Gormally & R Morgan, reproduced by permission of the literary executors.

decimal level into account?”³³ The notion of “decimal level” that she introduces here builds on the idea of reading the remarks step-wise, in the order that she had first proposed to von Wright, and indeed, 4.02 not only builds on 4.01 but in turn is part of a series on the same decimal level (4.03-4.06) that further discusses the connections between understanding a sentence and the picture theory. In other words, the two remarks quoted below are usually printed in number order, as part of a train of thought that is interwoven with two dozen other remarks, but when taken together, one can see that the second refers back to the first.³⁴

4.01 A sentence is a picture of reality.

A sentence is a model of reality as we imagine it.³⁵

4.02 This we can see from our understanding the sense of a sentential sign, without its having been explained to us.

There is more than one reason why this kind of reading of the *Tractatus* only began to be taken seriously relatively recently. First of all, Anscombe’s suggestion to von Wright was no more than an inspired conjecture, and one that she never made in print. Furthermore, as we have seen, it does take considerable effort and determination to read the *Tractatus* in the order Anscombe recommended if one has to start from the published text. Without a principled account of the use of the numbering system in the composition of the book, this alternate structure must have seemed

³³ Anscombe Archive, Box 5, File 186, p. 119. The Collegium Institute Anscombe Archive at the University of Pennsylvania, The Kislak Center for Special Collections, Rare Books and Manuscripts. © M C Gormally & R Morgan, reproduced by permission of the literary executors.

³⁴ See Mayer 1993, 115, for her discussion of the relationship between 4.01, 4.016 and 4.02. See also Hacker 2015, 652 and 660 for his discussion of this case and three other examples of anaphoric references in the *Tractatus* that are puzzling if taken as referring back to the previous remark in the number order, but can be explained as referring back to the previous remark in the tree order: (1) “now” in 4.5 follows on from 4.4, not 4.4661; (2) “here” in 5.4 connects up with 5.3, not 5.32; (3) “here” in 5.64 connects up with 5.63, not 5.634.

³⁵ Anscombe Archive, Box 5, File 186, p. 113. The Collegium Institute Anscombe Archive at the University of Pennsylvania, The Kislak Center for Special Collections, Rare Books and Manuscripts. © M C Gormally & R Morgan, reproduced by permission of the literary executors.

more like a striking coincidence or a happy accident than the product of the author's intentions.

Nevertheless, it seems very likely that "The Structure of the *Tractatus*"³⁶, the first chapter of Erik Stenius's *Wittgenstein's Tractatus: A Critical Exposition of Its Main Lines of Thought* (1964) was written in response to Anscombe's proposed alternate reading order.³⁷ While Anscombe's contribution to Stenius's book is only acknowledged in the most general terms in the preface, that first chapter is, in effect, devoted to an examination of her proposed method of reading the *Tractatus*.³⁸ After beginning the chapter with a brief consideration of the numbering system in which he contends that Wittgenstein not only did not follow his own rules about the numbering, "but does not keep consistently to any rule" (4), Stenius nevertheless devoted the rest of the opening chapter to seeing how far one can get by using the numbering system to "form a kind of general survey of the contents of the *Tractatus*" (5) along strikingly Anscombian lines. He first sets out some of the main connections between the seven whole-numbered remarks, observing that they can "be read as a connected whole and that they then express a continuous line of thought. ...the seven main theses in the *Tractatus* really form stages in the pursuit of a line of thought" (1964, 6-7). Next, Stenius suggests that it "would be tempting on the basis of this observation to try to continue reading the *Tractatus* by picking out the theses in the order given by the *emphasis* suggested through the numbering" (1964, 7). This leads him to a close reading of some of the main branches of the tree structure, namely those that are directly attached to the trunk formed by the whole-numbered remarks. He includes under this rubric not only 2.1-2.2, 3.1-3.5, ... 6.1-6.5, but also 2.01-2.06, 3.01-

³⁶ Stenius does not use italics for "Tractatus" here and elsewhere in his book, but I have followed the usual convention of italicizing the book title in my quotations from his writing.

³⁷ On the other hand, in his discussion of the relationship between 4.01, 4.016 and 4.02, Stenius takes it for granted that the number order determines the reference of "this" in 4.02, even as he acknowledges that it makes better sense to take it to refer to 4.01, the previous remark in the tree order: "The word 'this' refers formally to what was said in 4.016, but in fact the meaning of the statement becomes clearer if we let the word refer to what was said in 4.01" (Stenius 1964, 11).

³⁸ Stenius spent two months in Oxford on a British Council grant while working on the book, and explicitly acknowledged in the preface that he "profited from [his] discussion of different points with Miss G. E. M. Anscombe" (1964, xi) while he was there.

3.05 and so on, and finds that many of these main branches of the tree structure do offer “a more detailed expression of the line of thought indicated in the main theses” (14). On the other hand, Stenius also notes that the numbering system is not always employed in a strictly hierarchical way. Thus he observes that some of these series of remarks, such as 2.1-2.2, and 3.1-3.5, are better seen as “preambles”, or extended introductions to the topic of the next whole-numbered remark, while it is 2.01-2.06 and 3.01-3.05 that are the principal sets of “comments” on 2 and 3, the next level up in the tree order (Stenius 1964, 8-10.)

Finally, Stenius briefly considers taking the next step of moving on to the next level, or the “third degree” in his terminology—in other words, those branches that are attached to the main branches he has just considered—and looking at them as a “further expression of the line of thought expressed” (14). At this point, Stenius argues that not only is there nothing to be gained by going further in this direction, but that to do so would be seriously mistaken, for “to continue the analysis on the lines of the above principles would give a *wrong* picture of the structure of the *Tractatus*” (14).

In so doing, Stenius makes a very strong case against the view that the tree-structured reading is uniquely privileged when it comes to settling questions about the structure of the *Tractatus*, or Bazzocchi and Hacker’s view that the *Tractatus* “must be read [as] a logical tree.”³⁹ Stenius’s leading argument for this conclusion starts from a striking example: the role of the notion of “logical space” in the *Tractatus*. It is first explicitly mentioned in 1.13, “The facts in logical space are the world”. However, Stenius observes that as no further elucidation of the concept is provided in the 1s, we must turn to the relatively small number of other remarks in which “logical space” (or “space” used in roughly this sense) occurs if we are to work out what Wittgenstein meant by this term. Those remarks he

³⁹ See Bazzocchi (2014, 2014a, 2015, 2021) and Hacker (2021). Cf. Hacker 2015, 649: “The *Tractatus* must be read in accordance with the numbering system, and that demands that the reader follow the text after the manner of a logical tree”. Here Hacker may seem to be claiming that there is no alternative to a tree order reading, yet elsewhere on the same page he makes the weaker, and much more plausible, recommendation that one “avoids reading the work only consecutively, and also reads it tree-wise” (2015, 649). See also Kraft 2016, a reply to Hacker and defense of reading the *Tractatus* in number order, and Kuusela 2015, a critique of some of the methodological conclusions drawn by Bazzocchi and Hacker.

considers under this rubric are scattered through the 2s, 3s, and 4s, and if one considers them from a tree-structured perspective, they take the form of comments on a range of different concepts.⁴⁰ Stenius observes that we can “best characterize the function of 1.13 by saying that it is a first hint of a motif which weaves together different parts of the work into a fabric where everything is connected with everything else” (15). This leads him to the conclusion that “to understand the exposition in the *Tractatus* one must notice that on the one hand it *is* a continuous presentation whose structure we can study on the basis of the numbering; on the other hand, however, this continuous thread of thought is crossed by other threads in different directions which can be discerned on the basis of the recurrence of the motifs” (15).

Stenius’s explanation for the way in which the exposition of certain Tractarian themes, such as logical space, do not fit into the tidy hierarchical structure outlined in Wittgenstein’s opening footnote, and prized by readers, such as Bazzocchi and Hacker, who take the tree-structure to be the only structuring principle governing the arrangement of the book, is that it is simply impossible to fit and elucidate every separate key concept in tree fashion.⁴¹ The talk of “motifs” that form crisscrossing threads is part of a larger musical metaphor that he uses in order to contend that the book is animated by multiple inter-related themes that cannot be shoehorned into a series of hierarchical boxes.

On the other hand, in the light of subsequent discoveries about how the book was assembled, it is possible to offer a genetic account of the criss-cross discussion of logical space. The topic of logical space is not fully accommodated within the hierarchical tree-structure because it was only fully developed relatively late in the writing process and so did not neatly fit within the structure Wittgenstein had developed earlier on. From a broader perspective on this and related developments in Wittgenstein’s changing

⁴⁰ Stenius cites 2.013 and 2.0131 (“which... comment on the concept ‘thing’”); 2.11 (“which comments on the concept ‘picture’”); 3.4 and 3.42 (“which comment on the concept ‘sentence’”); 4.463 (“which comments on the concepts ‘tautology’ and ‘contradiction’”). See Stenius 1964, p. 15.

⁴¹ I want to thank Gabriel Citron for his suggestions about how best to understand this problem, and for pushing me to discuss this topic in more detail.

conception of his overall project, it is worth noting that a distinctive formal feature of the changes from *Prototractatus* to *Tractatus* is that in the 3s, 4s, and 5s Wittgenstein added new remarks to the sequences of single-decimal numbered remarks, extending those main branches of the tree structure by promoting remarks that had been at the two or three decimal level. Thus *Prototractatus* 3.3 became *Tractatus* 3.5. In between these remarks, Wittgenstein inserted a new 3.3 and 3.4, promoted from much less central positions in the previous arrangement (*Prototractatus* 3.202 and 3.2101). The new 3.3 and 3.4 serve as the point of departure for discussion of a topic that is consequently given greater prominence and significance, namely Frege's "context principle", that "Only sentences make sense; only in the context of a sentence does a name have meaning" (3.3 ff.). There is a rich, detailed and careful discussion of the importance of these changes for the book as a whole in Kremer (1997). On the other hand, while the promotion of *Prototractatus* 3.202 to *Tractatus* 3.3 led to a radical reorganization of many of the other remarks in the 3s under the new 3.3, only three remarks serve as comments on *Tractatus* 3.4 and discuss the idea of logical space (3.4 ff.)

When Wittgenstein began to write the book, he was still working within a broadly Russellian logical atomist "bottom-up" conception of meaning, one that is most explicitly stated in the pre-war "Notes on Logic". As Peter Hanks puts it, "on this approach, the connections between the elements of the proposition and things in the world take explanatory priority over the possession of sense by the whole proposition" (2014, 8). On this approach, it may seem as if there is no need for a notion of logical space, or that such a notion can be constructed from bottom up. However, it was only at a fairly late stage in the process of composing the *Tractatus* that Wittgenstein moved towards a "top-down" approach, on which, as Hanks puts it, "the correlations attaching to the components of the proposition are explanatorily posterior to the possession of sense by the whole proposition" (2014, 10).

Stenius's discussion of the way that "continuous presentation"—following the number order reading—is only part of the overall structure of the *Tractatus*, which includes a variety of other threads that connect parts of the text that are far apart in terms of their

placement strikes just the right balance between a tree-structured reading and a number order reading the *Tractatus*. On the other hand, thanks to our knowledge of the *Tractatus* source manuscripts, we are now in a position to better understand *why* the remarks about logical space are scattered through the text and not allocated a place of their own in a tree order system: most of them were added very late in the process of composition and so were grafted onto the structure that had already been put in place.

As we have just seen, there are Tractarian themes that are not presented by means of the numbering system. However, while this does make it clear that the structure of the *Tractatus* is more complicated than advocates of an entirely hierarchical picture would have us believe, it only goes to show a tree-structured reading cannot be the whole truth, but not that it is entirely “*wrong*”, as Stenius rashly claims. Indeed, the “picture of the structure of the *Tractatus*” that we find if we continue a tree-structured reading to the “third degree” and beyond, in Stenius’s terminology—that is, to those sequences of remarks containing two or more cardinal numbers after the decimal point, or the branches attached to the main branches—is still a valuable and informative one.

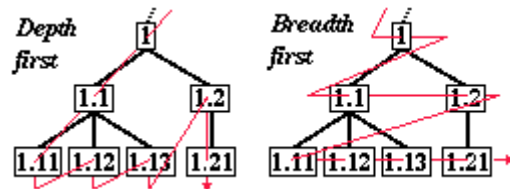
While our appreciation of certain themes can be guided and informed by the numbering system, we must also remain alert to other threads that are woven into that structure. My own view is that *both* of these approaches—a tree order reading, and a number order reading—are legitimate and appropriate interpretive strategies, and that to hold that either one of them is the *only* correct way to read the text is a mistake. In other words, we need to pay attention both to the number order in which the book was published, and the tree order that was used in composing and assembling earlier versions of the book.

In a note about his online hypertext edition of the *Tractatus*, Laventhol explains the rationale for his site in a section on “What order to read the propositions?”:

One [of] the many questions which the *Tractatus* raises is this: is really written to be read linearly? Does it make -- possibly more -- sense to read it in the other order? One of the unusual features of the book is

that it *has* an obvious other way to read it, ignoring browsing: read all the one-digit sections, then the two-digit sections, and so on.

The printed order we could call "depth-first", and the other order "breadth-first" -- following the conventional computing terms -- and can be shown like this:



The experiment is to read the text in breadth-first order, which is impractical with the printed book: too much page-turning.⁴²

While Laventhol's "obvious" alternative is very similar to Anscombe's, it is not quite the same. Like Anscombe, Laventhol proposes starting by reading all the one-digit remarks, but while he suggests next reading all the two-digit remarks, Anscombe had recommended reading them together with the one-digit remarks. There is a case to be made for each approach. Anscombe's recommended reading order appears to have been based on the discussion of the relationship of commenting in the second sentence of Wittgenstein's footnote to the first remark of the *Tractatus*.⁴³ That is, it seems to be designed in order to enable us to read "*n.1, n.2, n.3, etc.*" in order as "comments on remark number *n*"; "*n.m1, n.m2, etc.*" as "comments on remark number *n.m*; and so on." Laventhol's recommended reading order, on the other hand, takes its inspiration from the first sentence, which tells us, as Laventhol puts it, that Wittgenstein "puts more emphasis on the shorter numbers". Consequently, Laventhol suggests starting with the shortest numbers, and then working one's way through successive layers. This concentrates the reader's attention on the individual sequences, such as "*n.1, n.2, n.3, etc.*" and "*n.m1, n.m2, etc.*" but at the price of not attending to the relations between each sequence of remarks and the lower-numbered remark that the sequence comments on. Crucially, whether or not one ultimately considers Anscombe's

⁴² Laventhol 1996a.

⁴³ See page 3 above.

method, Laventhol's method, or some other way of combining depth and breadth preferable, the tree-structured edition that accompanies this essay, is quite well-suited to any of them, and a reader can easily experiment with different ways of navigating around the sequences of remarks that make up the *Tractatus* tree.

However, none of this shows that the *Tractatus* should only be read in tree order, or that the numerical order of the remarks on the printed page can be disregarded. In the end, the question of how best to read those remarks is one that can only be settled, passage by passage, by means of a close reading and evaluation of *all* the relevant texts. Anyone who wants to understand the *Tractatus* needs to read it equally carefully in number order and tree order. Those interested in its genesis, and the earlier versions of the book Wittgenstein composed while he was a soldier, should also take the time to read MS 104, not only in the number-order reconstruction provided in the *Prototractatus*, but also in both tree order and the page order in which it was originally written down. In the end, the strongest case for a tree-order reading is simply to read such an edition for oneself, and to see how each series of remarks both amounts to a complete and distinct unit, and at the same time functions as a comment on the connected remark at the next level up.⁴⁴

⁴⁴ Earlier versions of parts of this paper were presented in 2016 at the “Von Wright and Wittgenstein in Cambridge: von Wright Centenary Symposium”, held at Strathaird, Cambridge, UK, at a session on early analytic philosophy organized by the Society for the Study of the History of Analytic Philosophy at the American Philosophical Association's Central Division, held in Kansas City, via video link at the 9th Summer School on Mind and Language, organized by Luciano Bazzocchi at the University of Siena, Italy, the 2017 Kirchberg Wittgenstein Symposium, and a Russell Society meeting at the American Philosophical Association's Central Division, held in Chicago, in 2018. I learned a great deal from the discussion at all these events, and also from those who took part in my Fall 2015 and Spring 2022 graduate seminars at the University of Iowa and want to express my gratitude to everyone who took part.

I would like to thank the editors of *Nordic Wittgenstein Review* for inviting me to write this essay and especially Simo Säätelä for going above and beyond the call of duty in helping with the editorial work on the accompanying tree-structured editions of the *Tractatus*. I also want to thank Joachim Schulte and Katia Saporiti for our many conversations about this material, and their invaluable editorial work on the accompanying tree-structured text. I am particularly grateful to Gabriel Citron for our conversations about the manuscript sources of the *Tractatus*, and his detailed and extraordinarily helpful comments on an earlier draft of this paper. For further discussions of these topics and related matters, see Stern 2016, 2018, 2018a, and 2019, which include earlier versions of parts of the material in this paper.

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